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Application of a heuristic search technique for the improvement of spray zones cooling conditions in continuously cast steel billets

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Abstract

The process of casting occupies an important place in the metallurgical industry, and the entire world of the metal user. In the past, the ingot casting–rolling (slabbing, blooming, or billeting) process was commonly used. The continuous casting process has largely replaced this earlier method because of the inherent advantages of energy savings, enhanced productivity, higher yield and reduced costs. However, continuous casting process is not without of its problems. Considerable effort has been made by many researches to establish adequate design, operation and maintenance of continuous casting machine to ensure metallurgical quality of the final product. One of the most severe defects in continuous casting products is concerned with the cracks provoked by improper design of the spray cooling system. The aim of this work is to develop a two dimensional heat transfer model based on the finite difference method in order to calculate the strand temperatures and the solid shell profile along the machine. An Artificial Intelligence heuristic search procedure interacts with the numerical model to determine the improved cooling conditions for the sprays zones of a real continuous caster for the production of quality billets.

Keywords: Continuous casting; Mathematical modeling; Steel; Artificial intelligence

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1. Introduction

The process of casting occupies an important place in the metallurgical industry, and the entire world of the metal user. In the past, the ingot casting–rolling (slabbing, blooming, or billeting) process was commonly used. The continuous casting process has largely replaced this earlier method because it has better productivity and heat effectiveness.

In the continuous casting process, the liquid steel is constantly cast into semi-finished shapes, with the casting section encountering three entirely different cooling environments: a water-cooled mold, a series of water or air-mist sprays and the radiation zone before fully solidifying. The cooling process in the mold and spray zone removes the superheat, the latent heat of fusion at the solidification front and sensible heat from the solid shell. Fig. 1 shows schematically the continuous casting process.

For the sake of competitiveness in manufacturing, there is a permanent requirement of securing steel quality on steelmakers. In terms of process control, this means a necessity to maintain operational parameters in a specific optimum range. Owing to economic reasons, it is not feasible to undertake extensive experimental trials during the continuous casting process to evaluate the influence of several operational parameters. In this sense, the development of best operation in steel continuous casting is normally based on process simulations performed with heat transfer mathematical models.

In the literature, applications of heat transfer analysis to continuous casting using both analytical [1,2] and numerical [3–7] techniques have been presented. The execution of these models provides the thermal history of the cast under varying process variables to determine a set that results in defect free castings. However, considerable effort may be made, even if model simulation is applied, in search of this improved parameters set. By contrast, solving problems by heuristically guided, trial-and-error search in a space of possible solutions is a dominant theme in artificial

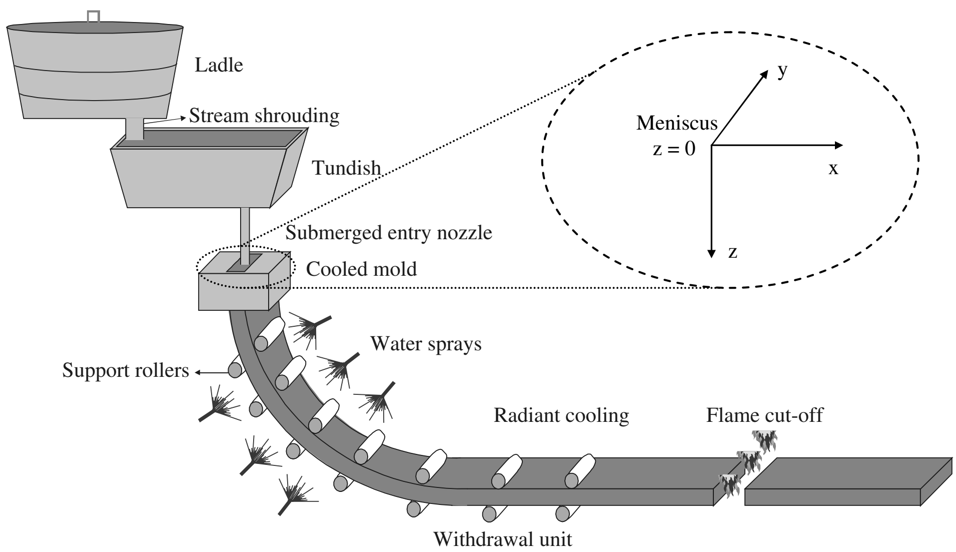

Fig. 1. Schematic representation of the continuous casting process.

intelligence (AI) field. Fundamental to the development of AI systems are a Knowledge Base which is obtained by the transformation of problem solving information from experts in the domain of application, into computer language statements.

A very effective work of Brimacombe, Samarasekera and their colleagues [8–12] on continuous casting has led to a fundamental knowledge of the links between cast quality and factors relating to casting and cooling conditions. As a result of this group extensive experience on continuous casting, an expert system for billet quality analysis was implemented [13,14]. Based on genetic algorithms search procedure, Filipic and Sarler [15] determined operational parameters to achieve the highest possible quality of the cast steel. Chakraborti and collaborators [16,17] developed a genetic algorithm to optimize the mould region in the continuous casting process, improving casting velocity for typical billet casters as a function of a number of process variables. Kominami et al. [18] developed a break out prediction system on the basis of neural networks technique. By using the same approach of neural networks, Zietsman et al. [19] investigated mold taper design. Dussud et al. [20] applied fuzzy logic to control the casting mold level through a designed fuzzy controller based on operators expert knowledge. Garcia and his co-workers [21–23] have used a heuristic search technique, knowledge base and a genetic algorithm coupled with mathematical solidification models for the optimization of casting velocity and quality of carbon steel billets and slabs.

This paper combines a numerical model of the billet continuous casting process with an artificial intelligence heuristic search technique linked to a Knowledge Base to find values that result in defect free billet production. The integrated system operates autonomously: the algorithm navigates through the state space of cooling parameters and invokes the simulator (mathematical model) to evaluate the parameter settings, while, given the parameter values, the simulator calculates the thermal field in the billet and assesses the Knowledge Base consisted of metallurgical quality criteria represented by constraints. The Knowledge Base is a rule-based system constituted of metallurgical constraints that relate process and cast quality. Technological constraints are also included into the Knowledge Base to incorporate caster design limitations. The simulator employs detailed models of heat transfer mechanisms and uses a two-dimensional finite difference method to calculate the thermal field. The heat transfer mechanisms include a library of thermal-physical properties for various steel grades, heat transfer to the mould, to the spray coolant in the secondary cooling zone and to the radiant zone. Improved cooling conditions of the new configuration are compared to the original one.

2. Rule-based knowledge representation (metallurgical criteria)

All AI systems get their power from the knowledge they embody by concentrating highly specialized knowledge on a particular domain. To become knowledge, information and the data must be encoded into a suitable form for direct use. A number of important issues has been proposed in knowledge representation, which can be divided into four categories of representation schemes: logical, network, structured and procedural (rule-based systems) [24].

The knowledge base for casting quality billets consists of metallurgical process constraints (if ... then ...... rules). These constraints were structured in terms of ingot thermal behavior, with cause and effect relationships drawn from the literature. A constraint satisfaction problem involves a set of variables, each of which has a domain of possible values. A solution to a constraint satisfaction problem specifies an assignment of values to variables that does not violate the constraints.

Centerline segregation or internal cracking can form if straightening or bending operations are carried out on a section with a liquid center. In addition, at the unbending point, surface temperature is required to be outside of the low ductility through in order to avoid transverse surface cracking. Larreq and Birat [25] report that the upper limit of the low ductility varies between 900 and 1100 °C according to steel composition, mainly nitrogen and niobium contents. Spim Jr. et al. [26] in their work used the value of 870 °C for a low carbon steel. The following criteria must therefore be obeyed:

$$T_c \leqslant T_s \tag{1}$$

$$T_{surf} \geqslant 870\ ^{\circ}\mathrm{C} \tag{2}$$

where T_c is the central temperature; T_{surf} the surface temperature and T_s is the solidus temperature.

Below the mold, pressure from the liquid steel within the solidified shell can give rise to inter-roll bulging, resulting in strains at the solidification front, which can cause cracks formation, and which are penetrated with solute enriched liquid (segregation) [8,25]. Prevention of this type of crack is achieved by a well set up spray system to produce a shell that can withstand the ferrostatic pressure. This shell behavior is related to surface temperatures below an upper limit reported by Lally et al. [27] as 1200 °C for billet casting. Thus, the constraint is stated as:

$$T_{surf} \leqslant 1200\ ^{\circ}\mathrm{C} \tag{3}$$

Halfway cracks are related to billet surface reheating along the caster [12]. The reheating is the result of a sudden decrease in the heat extraction rate from the surface as the billet moves from the mold to the sprays, from a spray zone to the next one or from the sprays to the radiation cooling zone (Fig. 2). As a consequence, the surface tends to expand and tensile stresses at the solidification front develop provoking hot tearing. The amount of permitted reheating which avoid halfway cracks depends on a number of factors. One of the most important factors is casting structure. Steel with a predominantly equiaxed structure can withstand the crack formation until reheat of 200 °C [3], while columnar structure is more conducive to cracks with lower surface

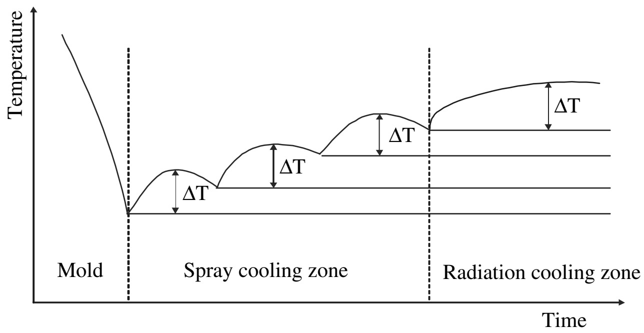

Fig. 2. Surface reheating along the different cooling zones.

Table 1
Knowledge based on rules

Region	Rule
Spray zone	If $T_{surf} > 1200$ °C then increase water spray flow If $T_{surf} < 870$ °C then reduce water spray flow If $\Delta T > 150$ °C then increase water spray flow
Unbending point	If $T_c > T_s$ then increase water spray flow

reheating. Lally et al. [27] in their paper have used the value of 175 °C for the maximum allowed surface reheat. In order to assure that the quality will be preserved, a maximum reheating of 150 °C has been adopted in the present work.

$$\Delta T \leqslant 150\,^{\circ}\mathrm{C} \tag{4}$$

where ΔT is the surface reheating in the spray and radiation cooling zones.

Table 1 summarizes the knowledge based on rules (if then structures).

3. Mathematical model

The numerical modeling of the strand has been developed to track a transverse slice of a steel billet as it moves down through the mold. The mathematical formulation of heat transfer is based on the fundamental equation of heat conduction [28]. Because of the high casting speeds and the high heat extraction rate at the surface (plane xz, yz), the axial conduction (z) of heat is negligible compared to the heat transferred by the bulkmotion of the strand. Therefore, heat conduction equation reduces to the two-dimensional unsteady conduction equation:

$$\rho \cdot c \cdot \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k(x) \cdot \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k(y) \cdot \frac{\partial T}{\partial y}\right) + \dot{q} \tag{5}$$

where k is the thermal conductivity [W/mK]; c the specific heat [J/kgK]; ρ the density [kg/m^3]; T the temperature [K]; $\dot{q}$ the rate of energy generation [W/m^3]; t the time [s] and x, y, z are the rectangular coordinates [m].

The billet symmetry permits that only one-quarter of the cross section be modeled for a full thermal evolution characterization. This transverse slice moves at the casting velocity through the different cooling zones of the machine. The release of latent heat between liquidus and solidus temperatures is expressed by $\dot{q}$:

$$\dot{q} = \rho \cdot L \cdot \frac{\partial f_S}{\partial t} \tag{6}$$

where L is latent heat [J/kg]. Substituting Eq. (6) into Eq. (5) gives

$$\rho \cdot c' \cdot \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k(x) \cdot \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k(y) \cdot \frac{\partial T}{\partial y}\right) \tag{7}$$

where c' can be considered as a pseudo-specific heat [29]:

$$c' = [(1 - f_S) \cdot c_L + f_S \cdot c_S] - L \cdot \frac{\partial f_S}{\partial T} \tag{8}$$

where the subscripts 'S' and 'L' refer to solid and liquid respectively and f_S corresponds to the local solid fraction. Convective heat transfer in the liquid pool is taken into account by increasing the thermal conductivity of the liquid steel in a factor of seven times [30].

Five boundary conditions are required to yield a solution for Eq. (7)

Time. At time = 0, the slice temperature profile at the meniscus ($z = 0$) is equal to the pouring temperature;

Billet center line. Considering symmetrical heat flow at the centerplanes ($x = X/2$, $y = Y/2$), heat does not flow across them (adiabatic);

Billet outer surface. The surface boundary condition represent the three zones of cooling which are characterized by the heat transfer coefficient 'h' [W/m^2 K] in each zone

$$x = 0, \quad 0 \leqslant y \leqslant \frac{Y}{2}, \quad -k\frac{\partial T}{\partial x} = h(T_{\text{surf}} - T_{\text{env}}),$$

$$y = 0, \quad 0 \leqslant x \leqslant \frac{X}{2}, \quad -k\frac{\partial T}{\partial y} = h(T_{\text{surf}} - T_{\text{env}}).$$

The heat transfer coefficient h in the spray zones is usually related to spray water flow rates. Approximate values can be found in literature [31], but for a specific caster and steel grade, these can be determined more precisely on the basis of surface temperature measurements, for instance, using pyrometers. A list of correlation between h and water flow rate for commercial nozzles is shown in Ref. [31]. The present work deals with the heat transfer coefficient h instead of the water flow in order to avoid particularization of the determined cooling conditions for only a type of nozzle.

The heat transfer equation and the set of boundary conditions were implemented via an explicit finite difference procedure. An explicit finite difference form of Eq. (7) is obtained as follows:

$$\begin{aligned} T_{i,j}^{n+1} = \frac{\Delta t}{(\rho \cdot c')_{i,j} \cdot \Delta x^2} \Big[& k_{\text{eq}_{i+1,j}}^n \cdot (T_{i+1,j}^n - T_{i,j}^n) + k_{\text{eq}_{i-1,j}}^n \cdot (T_{i-1,j}^n - T_{i,j}^n) + k_{\text{eq}_{i,j+1}}^n \cdot (T_{i,j+1}^n - T_{i,j}^n) \\ & + k_{\text{eq}_{i,j-1}}^n \cdot (T_{i,j-1}^n - T_{i,j}^n) \Big] + T_{i,j}^n \end{aligned} \tag{9}$$

where 'n' and '$n + 1$' refer to temperatures before and after the incremental time interval Δt; 'i' and 'j' are, respectively, the element position according to 'x' and 'y' axes and k_{eq} is the equivalent thermal conductivity in terms of the thermal conductivity of a adjacent element and itself.

4. Exploration of alternatives

Search is one of the most ubiquitous programming and problem-solving paradigm in artificial intelligence. Every AI algorithm involves search, whether by explicitly constructing and traversing a state space implicitly organizing its reasoning as a search among alternatives [32]. Search in a state space is a systematic exploration of the space in order to find one or more solutions that have specified properties. The aim of this section is to propose a heuristic search method that is efficient

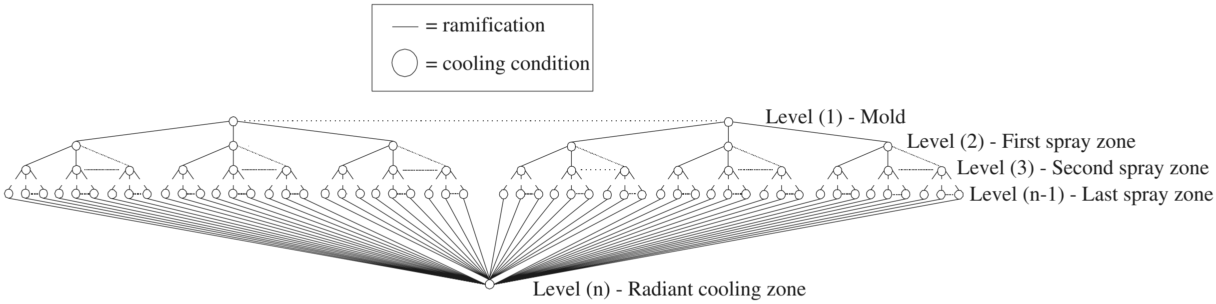

Fig. 3. Representative tree of cooling conditions.

in finding improved cooling condition sets from an extensive number of possible alternatives, which can be represented by a tree scheme (Fig. 3). The tree has 'n' levels corresponding to the number of continuous casting machine cooling zones, where the first level is the mold and the last (n) is the radiant cooling zone. Between the first and last levels, ($n-2$) levels characterize the spray cooling zones. As shown in Fig. 3, each node represents a cooling condition and is linked to subsequent nodes of possible solutions, generating an explosively combinatorial expansion.

One way to minimize the search is to state principles or rules of thumb to help reduce the search. Any technique used to speed up the search depends on special information about the problem being analyzed. For this reason, a well structured Knowledge Base is important to any AI systems. Otherwise, no matter what search method is used, the search would be infeasible due to the combinatorial node explosion. In other words, the more "informed" the Knowledge Base is, the less extraneous successors nodes are generated.

The literature presents many search methods such as: depth-first search, breadth-first search, hill climbing, beam search, nondeterministic search, best first search, backtracking strategy and others [33–36]. The method used in this work is the beam search with the rules described in the Knowledge Base section. The search progresses level by level only through the nodes that satisfy the rules and prunes the branches that violate the constraints. Consequently, the explosive number of nodes is avoided.

A technological constraint of the spray cooling system may be considered into the AI system that is the hydraulic circuit control limitation. In terms of mathematical modeling and metallurgical quality concerning to halfway cracks, the best spray design is a smooth decrease in water flow rate represented by an equation that describes the h variation along the spray cooling zone. Due to control limitations it is not feasible to build this type of spray system. Therefore, the best solutions are the ones where the theoretical results best fit the operational and design constraints (Fig. 4). For this reason, steps of 50 W/m^2 K have been applied between the maximum and the minimum flow rates. Steps values lower than 50 W/m^2 K do not change considerably the billet thermal profile.

The resultant improved cooling conditions furnished by computer simulations consist in a set of cooling conditions which satisfy the reheating and cooling limits imposed by metallurgical and caster machine criteria and in addition, these conditions perform minimum metallurgical length.

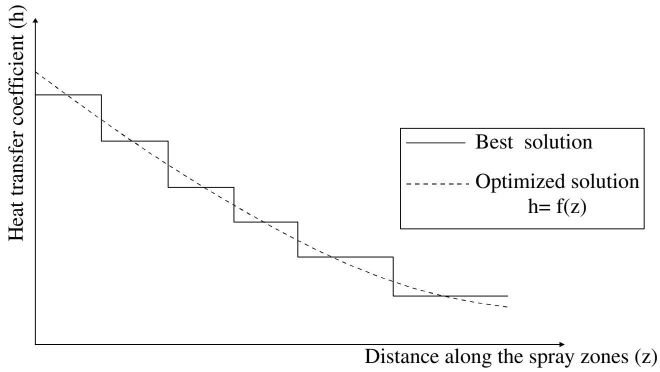

Fig. 4. Comparison between best and optimized solutions.

5. Simulation results and discussion—optimization of a real billet caster

The above described strategy in terms of process parameters setting was applied to the real dimensions of a steel billet continuous caster. Simulations were carried out for a SAE 1018 carbon steel billet with a cross section of 0.15×0.15 m. The heat transfer coefficients were limited to minimum (h_{min}) and maximum (h_{max}) values as indicated in Table 2. The number of possible parameters settings is determined by multiplying the number of h values over the three spray cooling zones. By using the step of 50 W/m^2 K variation in each cooling zone, there are about 3×10^4 possible settings to explore.

The heuristic search results are listed in Table 3, which presents the cooling parameters that attain the best conditions to complete solidification in a minimum time. Table 3 also presents the position below the meniscus where the solidification ends i.e., the length of liquid pools also

Table 2
Dimensions, cooling limits in each cooling zone and other operational parameters

Mold		
Length: 0.7 m	Meniscus length: 0.1 [m]	$h = 1900$ W/m^2 K
First Spray zone		
Length: 0.615 m	$h_{max} = 1850$ W/m^2 K	$h_{min} = 200$ W/m^2 K
Second spray zone		
Length: 1.355 m	$h_{max} = 1800$ W/m^2 K	$h_{min} = 200$ W/m^2 K
Third spray zone		
Length: 4.230 m	$h_{max} = 1750$ W/m^2 K	$h_{min} = 200$ W/m^2 K
Radiant cooling zone		
Length: 4.6 m	$h = 150$ W/m^2 K	Environmental temperature: 100 °C
Pouring temperature: 1530 °C		
Casting rate: 0.023 m/s		
Cooling water temperature: 20 °C		
Metallurgical length: 11.4 m		

Table 3
Sets of best cooling conditions which provide the four shortest metallurgical lengths of all analyzed alternatives

	Set (1)		Set (2)		Set (3)		Set (4)	
	h [W/m^2 K]	ΔT [°C]	h [W/m^2 K]	ΔT [°C]	h [W/m^2 K]	ΔT [°C]	h [W/m^2 K]	ΔT [°C]
First spray zone	1100	74	1200	56	1050	83	1150	65
Second spray zone	800	63	750	63	800	68	750	54
Third spray zone	350	137	350	123	350	136	350	123
Metallurgical length	10.65 [m]		10.68 [m]		10.70 [m]		10.72 [m]	

Table 4
Sets of best cooling conditions which provide the four shortest metallurgical lengths of all analyzed alternatives—new spray configuration

	Set (5)		Set (6)		Set (7)		Set (8)	
	h [W/m^2 K]	ΔT [°C]	h [W/m^2 K]	ΔT [°C]	h [W/m^2 K]	ΔT [°C]	h [W/m^2 K]	ΔT [°C]
First spray zone	1200	58	1150	67	1200	58	1100	74
Second spray zone	900	30	900	32	900	30	900	36
Third spray zone	750	15	750	15	700	26	750	16
Fourth spray zone	550	38	550	38	550	36	550	37
Fifth spray zone	300	93	300	93	300	92	300	92
Metallurgical length	9.97 [m]		10.01 [m]		10.02 [m]		10.05 [m]	

known as metallurgical length. A reduction of about 700 mm on the metallurgical length is attained when compared to the solid shell profile furnished by cooling conditions of Set (1).

A new configuration for the secondary cooling zone is proposed by equally dividing the second and the third spray zones into two parts, resulting into five spray zones instead of three. Improved cooling conditions of the new configuration are obtained using the presented heuristic search ap-

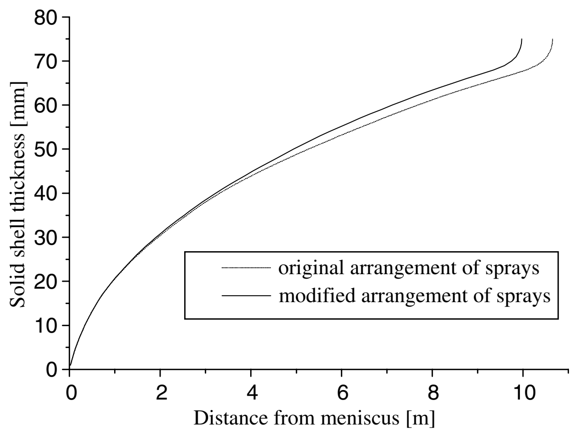

Fig. 5. Solid shell evolution according to cooling conditions of Sets (1) and (5).

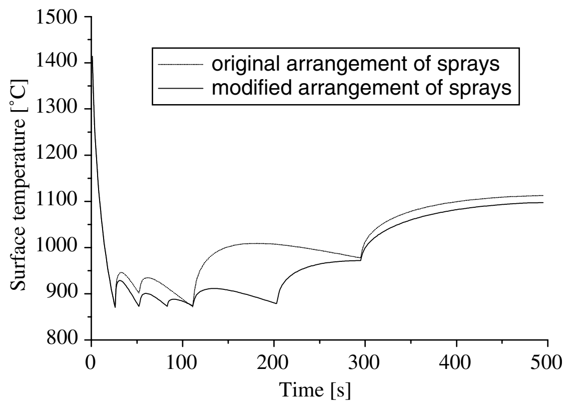

Fig. 6. Billet surface temperature for Sets (1) and (5).

proach. Table 4 shows the resulting best conditions and it can be realized that the metallurgical length obtained from the best set (Set 5) is 680 mm shorter than the one obtained from the original spray configuration (Set 1). Fig. 5 shows the evolution of billet solid shell thickness by using condition given by Sets (1) and (5).

Using Sets (1) and (5) of cooling conditions from Tables 3 and 4, surface temperatures ($x = X/2$, $y = 0$ or $x = 0$, $y = Y/2$) were plotted in Fig. 6.

From the discrepancy of behavior between the two sets, one can observe that Set (5) presents less reheating surface temperature than Set (1), which implies in less susceptibility to crack formation. The two additional spray zones for the new secondary cooling region has provided a better flexibility than the original one ensuring better billet quality.

6. Conclusion

The improved cooling conditions of mold and spray cooling zones of a continuously billet casting machine were determined with the application of a computational program based on an interaction between a heat flow mathematical model and an heuristic search technique supported by a Knowledge Base based on metallurgical quality and technological constraints. A two-dimensional finite-difference mathematical model has been developed and employed to predict the temperature field and pool profile of continuously cast steel billets. The heuristic search method, aided by the Knowledge Base, explores the space parameter settings in order to find improved cooling conditions which results in defect free billet production such as centerline segregation, internal cracking, transverse surface cracking, cracks at the solidification front, halfway cracks and break-out prevention. The heuristic search was shown to provide the best cooling parameters for completion of solidification in a minimum time what suggests a design modification by reducing the machine metallurgical length, or rather, the unbending point. A modification of the secondary cooling zone is proposed by adding two spray zones resulting in a shorter metallurgical length and lower surface reheating compared with the set of original spray zones. The developed AI search is useful as a method to improve caster performance and to control billet quality.

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