ISSN 0102-9924

FACULDADE DE CIÊNCIAS ECONÔMICAS DA UFRGS

Análise Econômica

ANO 25

Nº 47

Março, 2007

Prof. Sérgio Marley Modesto Monteiro

REVISTA ANÁLISE ECONÔMICA - Av. João Pessoa, 52
CEP 90040-000 PORTO ALEGRE - RS, BRASIL
Telefones: (051) 3308 3513 /3308 4164
Fax: (051) 3308-3990
Email: rae@vortex.ufrgs.br

Assinatura revista Análise Econômica: R$50,00
A assinatura anual dá direito a 2 números da revista.

Análise Econômica
Ano 25, nº 47, março, 2007 – Porto Alegre
Faculdade de Ciências Econômicas, UFRGS, 2007

Periodicidade semestral, março e setembro.
ISSN 0102-9924

1. Teoria Econômica – Desenvolvimento Regional - Economia Agrícola – Pesquisa Teórica e Aplicada – Periódicos. I. Brasil.
Faculdade de Ciências Econômicas,
Universidade Federal do Rio Grande do Sul.

CDD 330.05
CDU 33 (81) (05)

A Non-Parametric Test of the Solow-Swan Growth Model

*Adalmir Marquetti**

Resumo: Este artigo utiliza regressão local, um método não paramétrico, para testar o modelo de Solow-Swan em relação aos modelos de crescimento endógeno. A visualização da função de produção agregada revela a presença de retornos marginais decrescentes ao capital, em particular, para países com baixa relação capital-trabalho. A estimativa por regressão local da taxa bruta de lucro mostra que essa declina com o aumento da relação capital trabalho. Apesar dos resultados favorecerem o modelo de Solow-Swan, há indicações de que os modelos de crescimento endógeno representam uma boa aproximação para os países de elevada relação capital trabalho.

Palavras-chave: crescimento econômico, métodos não-paramétricos, função de produção.

Abstract: This paper employs a non-parametric method called local regression to test the Solow-Swan model against the endogenous growth models. The visualization of the aggregate production function reveals the presence of diminishing returns to capital, in particular, at low level of capital labor ratio. However, the evidence did not contradict the models of endogenous growth in which the marginal productivity of capital is bounded from below. The local regression fit of the gross profit rate showed that it declined as the capital labor ratio increased. Overall, the results support the Solow-Swan model, but indicate that the endogenous growth models are a close approximation for countries with high capital labor ratio.

Keywords: economic growth, non-parametric statistics, convergence, production function.

JEL Classification: C14; E20; O40.

Introduction

As it is well known the theoretical and empirical research in economic growth has experienced a new boom since the second half of the 1980s with the appearance of the Endogenous Growth Theory (EGT) and data sets in the form of panel data and long-term historical

* Ph.D. em Economia, New School for Social Research, EUA. Professor do Programa de Pós-Graduação em Economia da PUC-RS e Pesquisador do CNPq. Email: aam@pucrs.br

Recebido em maio de 2006. Aceito em maio de 2007.

data. The growth in per capita income is explained within the model through the elimination of diminishing returns to capital in the EGT. At the empirical level the question that attracted greater attention is whether per capital income in different countries is converging.

The assumption of diminishing returns to capital implies in the concavity of aggregate production function in Solow-Swan growth model. The concavity of the aggregate production function might result in a long run convergence among poor and rich countries. The unconditional convergence hypothesis implies that, if all countries have access to the same technology represented by an aggregate production function with constant returns to scale, have the same savings rate and growth rate of population, then capital and income per capita will converge to the same steady state level in which the growth rate in per capita income is explained by the exogenous technical change. The conditional convergence hypothesis implies in the possibility of different steady states between countries due to differences in saving rates and population growth.

The abandonment of the assumption of diminishing returns leads to the rejection of convergence by most of the EGT literature. The debate over the existence of convergence has been investigated mainly with parametric tests assuming previous functional forms and distribution functions.

Kurz and Salvadori (1997) classify the EGT models in three groups based on the mechanisms "embodied" in the aggregate production function that eliminate the diminishing returns to capital or the tendency for the profit rate to decline as the capital labor ratio increases. The first group represented by Rebelo (1991), King and Rebelo (1990), and Young (1992) assumes constant returns to capital, the AK models. The second group represented by Jones and Manuelli (1990) assumes that diminishing returns to capital are bounded from below, the combination of an AK with a traditional neoclassical production function. Models in which the aggregate production function may present convex regions represent the third group. This group is split in models with externalities represented by Romer (1986) and Lucas (1988) and in models with scale effects in growth represented by Romer (1990), Grossman and Helpman (1991) and Aghion and Howit (1992).

Thus, the shape of the aggregate production function provides the necessary information to test the exogenous growth model against the endogenous models, except for Jones and Manuelli (1990). It also allows distinguishing between the three groups of models within the EGT literature. The Solow-Swan growth model contrasts with the Jones and Manuelli (1990) by the tendency of the economy to move to its steady state. Moreover, for Jones and Manuelli (1990) the production function must become linear and the elasticity of output with respect to capital must be equal to one at some level of the capital labor ratio.

The purpose of this paper to test the Solow-Growth model against the endogenous growth models employing a non-parametric method named local regression (CLEVELAND, 1993, and LOADER, 1999). Local regression allows us to visualize the full representation of the Solow-Swan growth model and the elasticity of output with respect to capital. The advantages of this method are that no parametric functional form is previously assumed and that it allows us to compute derivatives. Furthermore, visualization is a powerful mechanism for data analysis and communication. It allows seeing aspects that are not possible in the parametric framework. The evidence in support of the various growth models will be examined by looking at data on labor productivity and capital labor ratio for 108 countries in 1985 expressed in 1996 purchasing power parity (PPP) (HESTON, SUMMERS, ATEN, 2002, and our net standardized estimates of the fixed capital stock). The number of observations for 1985 is greater than for other years. The data set is available on request.

1 The Solow-Swan growth model

The basic structure of graphical presentation will follow the Solow-Swan growth model. The aggregate production function takes the form $X_t = F(K_t, A_tL_t)$, where X is the flow of output, K is tangible capital, A is an index of technology, L is labor input, and t represents time. The exogenous technical progress takes the form of Harrod-neutral technical change. The AL term is a measure of the effective labor inputs. The production function is assumed to have constant returns to scale, thus it can be written in intensive form as $\tilde{x}t = f(\tilde{k}t)$, where $\tilde{k}$ is the capital per effective labor ratio and $\tilde{x}$ is the output per effective labor. Furthermore, the production function has positive and diminishing marginal products with respect to labor and capital, and satisfies the Inada conditions. From the assumptions of the Solow-Swan model, only constant returns of scale is necessary to assume to perform our statistical procedure.

The indexes of technology and labor inputs grow exponentially at cons ant exogenous rates g and n, respectively. Output can be consumed or saved. The share of output that is saved, s, is determined exogenously. By the condition of equilibrium in the goods market, St = It = Kt = sXt, and by the aggregate production function the fundamental differential equation of the Solow-Swan model can be written as $\dot{\tilde{k}}t = sf(\tilde{k}t) - (n + g + d)\tilde{k}t$, where d denotes the constant depreciation rate and the dot over the variable differentiation with respect to time. This equation states that the variation in the capital per effective labor ratio, $\dot{\tilde{k}}$, is determined by the difference between the amount of investment per unit of effective labor and the amount of investment that must be done to keep the capital per effective labor ratio constant, the break-even investment. The steady state $(\tilde{k}^*, \tilde{x}^*)$ is determined by the interception between the investment per unit of effective labor and the break-even investment. The Solow-Swan growth model is illustrated in Figure 1.

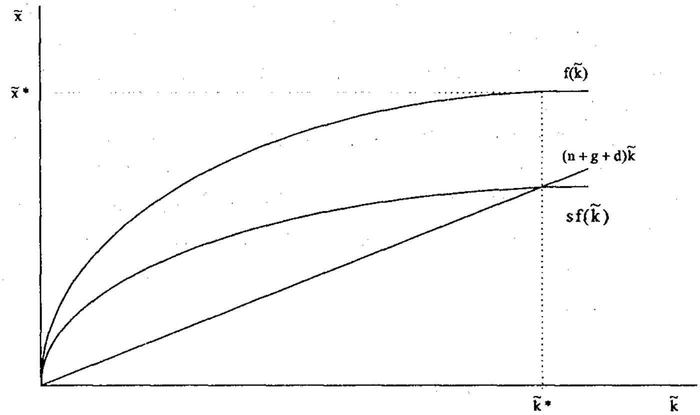

Figure 1. The representation of the Solow-Swan growth model. The $f(\tilde{k})$ curve is the production function. The $sf(\tilde{k})$ curve shows the savings per effective labor. The $(n + g + d)\tilde{k}$ line shows the break-even investment. The steady state $(\tilde{k}^*, \tilde{x}^*)$ is determined by the interception between $sf(\tilde{k})$ and $(n + g + d)\tilde{k}$.

In a model of perfect competition if all factors of production are paid according their marginal products, the elasticity of output with respect to capital is equal to the capital share. Three aspects related to the capital share are relevant in the discussion of our results. First, it can be demonstrated that $\hat{\tilde{x}} = \alpha\hat{\tilde{k}}$, were α is the capital share, and the hat over a variable denotes the growth rate. Hence, when the capital share is constant, the Cobb-Douglas case, or decline with the increase of the capital per efficiency labor units, we have convergence in output per effective labor since the Solow-Swan model the growth rate of capital per effective labor ratio in poor countries is higher than in rich ones. Second, the elasticity of output per unit of effective labor at the steady state with respect to saving, $\frac{d\tilde{x}^*}{ds}\frac{s}{\tilde{x}^*}=\frac{\alpha^*}{1-\alpha^*}$, depends on the capital share on the steady state. A high value of α^* implies that a change in savings rate has a large impact on output. Third, the rapidity of the convergence process to the steady state also depends on the value of the capital share. For similar parameters s, n, g, and d in poor and rich countries the speed of convergence, $\beta = (1 - \alpha)(n + g + d)$, is a function of the capital share. High values of capital share produce a slow process of convergence while low values produce a rapid process of convergence. Thus, the size of the capital contribution to output has played a major role in the convergence debate.

2 The representation of the Solow-Swan growth model

The remarkable comprehensiveness of the Penn World Table data allows us to examine the representation of the Solow-Swan growth model. The Penn World Table data is complemented with an estimation of the net standardized fixed capital stock. The methodology for calculating the net standardized fixed capital stock and the data treatment employed in the paper are described in Appendix 1.

Initially we consider the non-parametric estimate of the growth rate of Harrod-neutral technical change, saving rate, and growth rate of labor force. Loader (1999) and Cleveland and Loader (1996) present the basic idea as well as the mathematical details of local regression. These estimates will be employed in the representation of the Solow-Swan model. Thus, g, s, and n are allowed to change, reflecting the heterogeneity of the exogenous parameters. Most empirical literature on convergence computes g, s, and n as a time average.

Figure 2 plots the local regression fit for the (g, k) pair and the 95 percent confidence interval. Appendix 2 discusses some aspects of local regression. The growth rate of Harrod-neutral technical change was computed according to $g = \hat{x} - \alpha\hat{k}$, where α represents the capital share, $\hat{x}$ and $\hat{k}$ the five-year growth rates of labor productivity and capital-labor ratio. The goal of employing five-years growth rates is to reduce the effects of the business cycle over the estimated variables. There is data on capital share for 70 countries, a was measured as a time average of the available information in the period 1980-1990. The capital share for individual countries is relatively constant over time.

The local regression plot reveals three important aspects about g. First, countries with higher capital-labor ratio tend to have larger g. Second, it has negative segments for countries with capital per effective labor ratio below $ 35000 1996 PPP. Third, it approaches 0.9 percent as the capital per effective labor ratio increases. Mankiw, Romer, and Weil (1992) consider g equal to two percent.

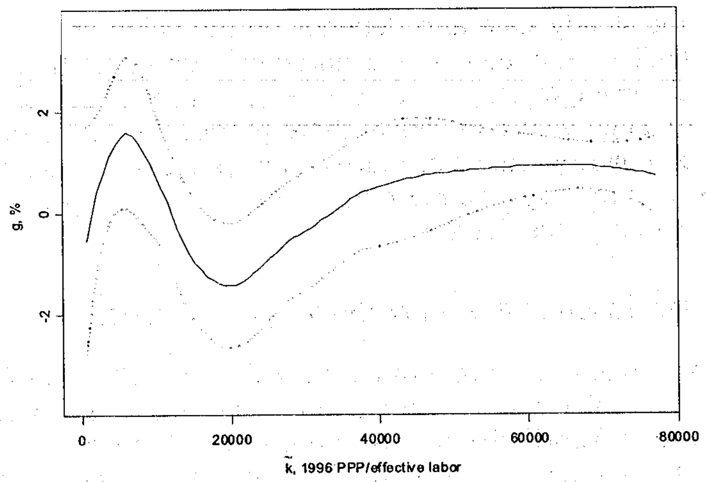

Figure 2. The local regression plot for the (k, g) pair, (HESTON, SUMMERS and ATEN, 2002, and our estimation of the net standardized fixed capital stock. Local regression parameters: bandwidth = 0.55, degree = 2, number of observations = 70).

Figure 3 presents the local regression plot for the (n, x) pair and the 95 percent confidence interval. It is possible to observe the demographic transition in the plot, n increases for low labor productivity countries and, then, declines as x increases approaching 0.9 percent.

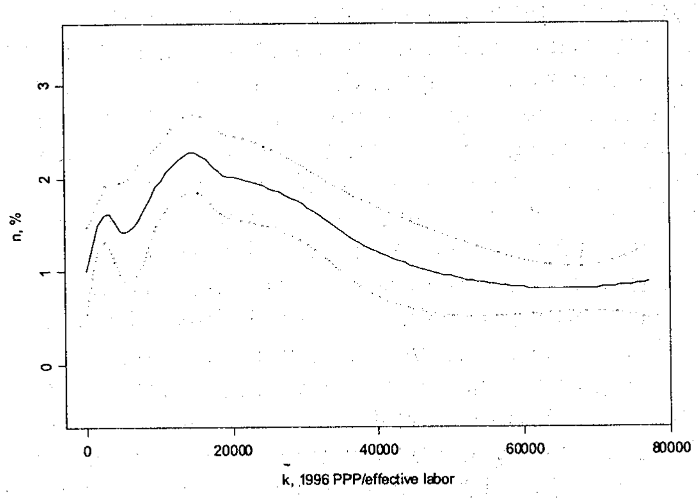

Figure 3. The local regression plot for the (x, n) pair, (HESTON, SUMMERS and ATEN, 2002, and our estimation of the net standardized fixed capital stock. Local regression parameters: bandwidth = 0.45, degree = 2, number of observations = 108).

Figure 4 shows the local regression plot for the (s, x) pair and the 95 percent confidence interval. The saving rate raises and approximates 23 percent with the increase in the capital per effective labor ratio.

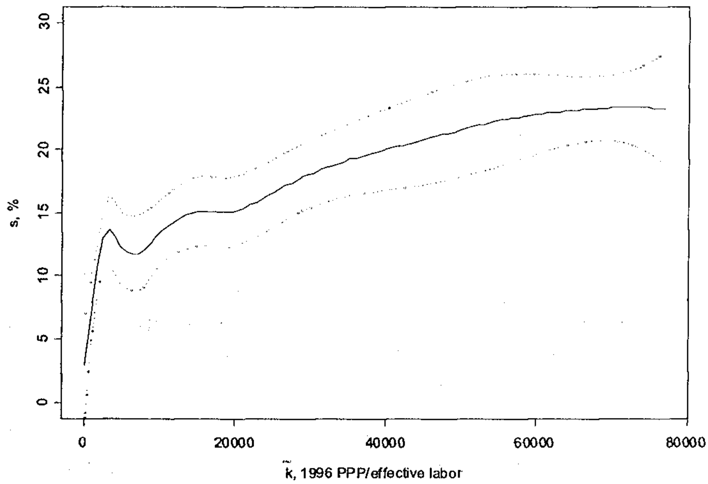

Figure 4. The local regression plot for the (x, s) pair, (HESTON, SUMMERS and ATEN, 2002, and our estimation of the net standardized fixed capital stock. Local regression parameters: bandwidth = 0.7, degree = 2, number of observations = 108).

Figure 5 plots the estimated aggregate production function from the $(\tilde{k}, \tilde{x})$ observations. It has a concave shape, reproducing a textbook production function from the data without assuming a previous functional form. However, we observe a more pronounced curvature at low levels than at higher levels of capital per effective labor ratio. In fact, for the capital per effective labor ratio above \$ 25000 1996 PPP a pattern very close to linear seems to be present in the estimated production function.

Therefore, the plot of the aggregate production function is inconsistent with the EGM with linear technologies, the AK models, as well as the EGM that may present increasing returns. The aggregated production function presents diminishing returns to capital, in particular, for low levels of capital per effective labor ratio. This result is consistent with Solow-Swan model as well as with Jones and Manuelli (1990).

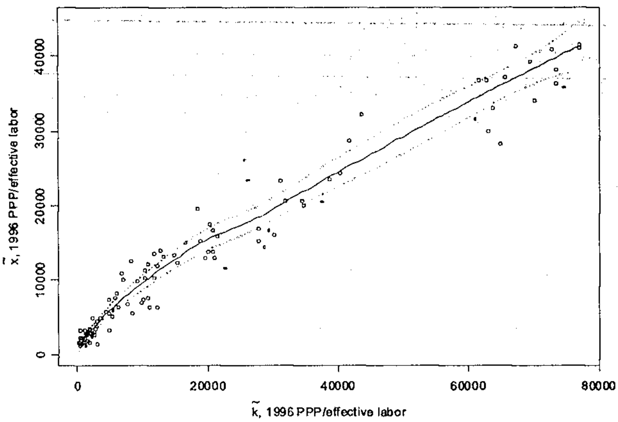

Figure 5. The estimated aggregated production function and the 1985 sample of $(\tilde{k}, \tilde{x})$ points, (HESTON, SUMMERS and ATEN, 2002, and our estimation of the net standardized fixed capital stock. Local regression parameters: bandwidth = 0.675, degree = 2, number of observations = 108).

The Solow-Swan and Jones and Manuelli (1990) models can be differentiated by the presence of diminishing or constant returns to capital at high capital per effective labor ratio. For the Solow-Swan growth model the aggregate production function is strictly concave due to the diminishing returns to capital, while for the Jones and Manuelli (1990) model it becomes linear due to the presence of constant returns.

Figure 6 plots the estimated marginal product of capital computed as the derivative of the fitted production function. It is negatively correlated with the capital per effective labor ratio as expected by the Solow-Swan model, declining from 1.1 to 0.45. However, the hypothesis that $f'(\tilde{k})$ is constant for a capital per effective labor ratio above $ 25000 1996 PPP cannot be rejected. This result is consistent with Jones and Manuelli (1990).

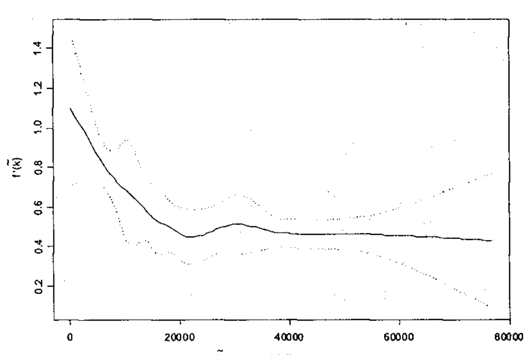

Figure 6. The marginal product of capital. It was computed as the local slope of the estimated aggregated production function in relation to capital.

Figure 7 shows the full representation of the Solow-Swan growth model and Figure 8 the sf($\tilde{k}$) curve and the (n + g + d)$\tilde{k}$ line. The steady state is not reached in the estimated interval, but the extrapolation of both fits show that it would happen at the point (120000 1996 PPP, 55500 1996 PPP).

There are interesting aspects in Figures 7 and 8. First, the sf($\tilde{k}$) curve is close to the (n + g + d)$\tilde{k}$ line and, second, the "almost" linearity of the sf($\tilde{k}$) at high capital per effective labor ratio. Thus, a small change in investment would have a considerable effect in steady state level of output per effective labor. This result is consistent with de Long and Summers (1992) who point out to fixed capital formation in the form of equipment investment as a key factor to economic growth.

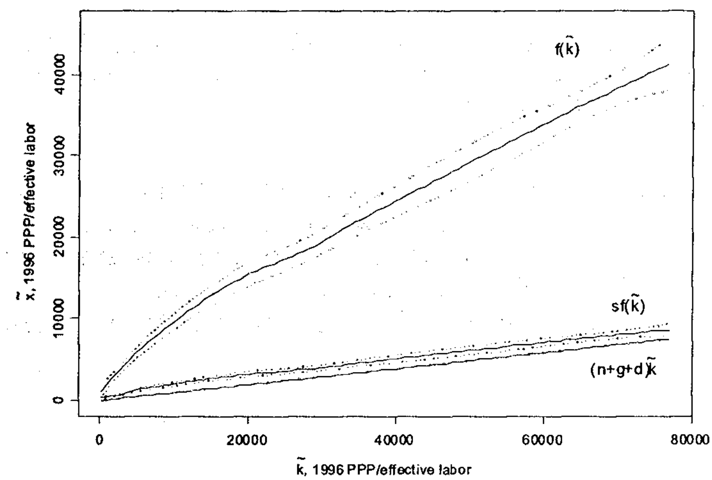

Figure 7. The local regression plot of the full Solow-Swan growth model, (HESTON, SUMMERS and ATEN, 2002, and our estimation of the net standardized fixed capital stock. Local regression parameters for sf($\tilde{k}$): bandwidth = 0.765, degree = 2, number of observations = 108. Local regression parameters for (n + g + d)$\tilde{k}$: bandwidth = 0.3, degree = 1, number of observations = 108).

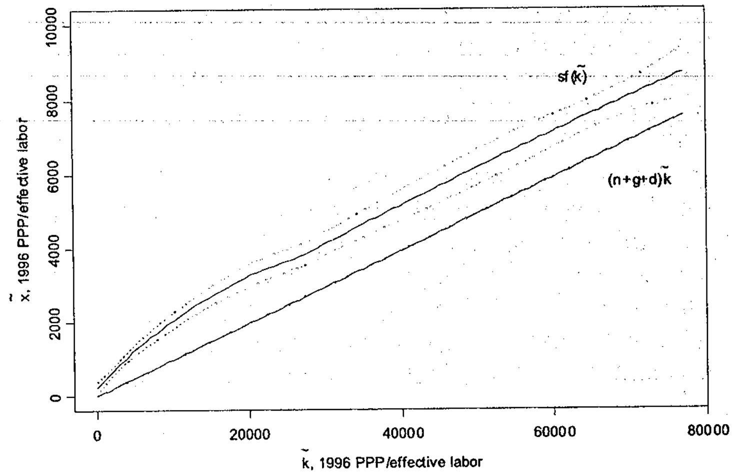

Figure 8. The local regression plot for the $sf(\tilde{k})$ curve and the $(n + g + d)\tilde{k}$ line (HESTON, SUMMERS and ATEN, 2002, and our estimation of the net standardized fixed capital stock).

Figure 9 plots the empirical estimation of the phase diagram of the Solow-Swan growth model. It was estimated considering the pair $(\tilde{k}, \dot{\tilde{k}} = sf(\tilde{k}) - (n + g + d)\tilde{k})$. The local regression fit is consistent with our previous results. By observing the plot, it is not possible to distinguish between the Solow-Swan model and Jones and Manuelli (1990).

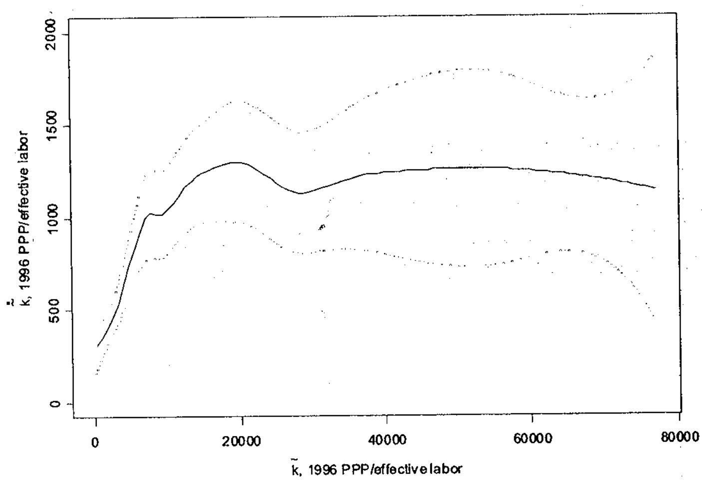

Figure 9. The local regression plot for the phase diagram of the Solow-Swan growth model, (HESTON, SUMMERS and ATEN, 2002, and our estimation of the net standardized fixed capital stock. bandwidth = 0.75, degree = 2, number of observations = 108).

Loader (1999, p. 172) presents the maximal deviation test, which allows testing the null hypothesis that the local regression estimate is constant. According to Jones and Manuelli (1990) the term $\tilde{k} = sf(\tilde{k}) - (n + g + d)\tilde{k}$ must become constant at high values of $\tilde{k}$. Thus, the maximal deviation test is an attempt to test Jones and Manuelli (1990) against the Solow-Swan model. Figure 10 shows the local slope estimate and confidence bands for the phase diagram of the Solow-Swan model. For capital per effective labor ratio above 55000 1996 PPP the null hypothesis that the local regression is constant cannot be rejected at five percent level of significance. This result is consistent with Jones and Manuelli (1990).

In short, the plots of the estimated production function and marginal productivity of capital support the presence of diminishing returns to capital, in particular at low levels of capital per effective labor. The results contradict endogenous growth models with constant and increasing returns to capital. On the other hand, they were dubious in relation to the Solow-Swan model and Jones and Manuelli (1990).

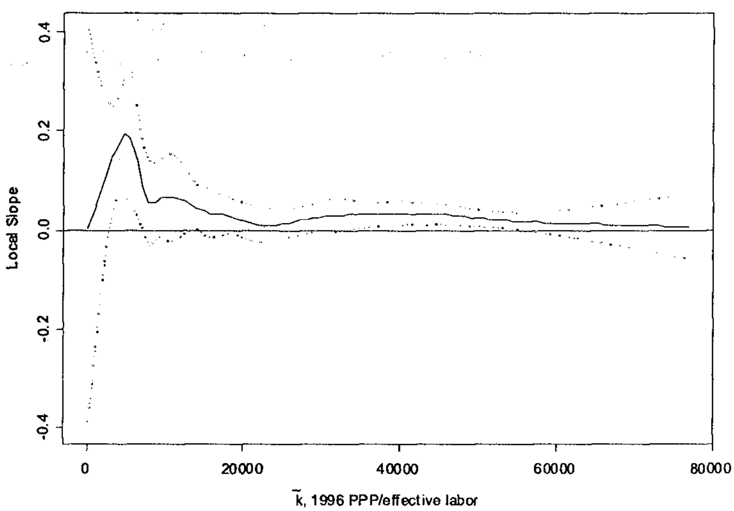

Figure 10. Local slope and confidence bands for the phase diagram of the Solow-Swan model.

3 The pattern of the elasticity of output with respect to capital

An attempt to distinguish between the Solow-Swan model and the Jones and Manuelli (1990) is to investigate the pattern of the elasticity of output with respect to capital. In the Jones and Manuelli (1990) model as the capital per effective labor ratio increases, α should approximate to one so that the diminishing returns vanish and constant returns to capital prevail in the long run.

The elasticity of output with respect to capital is obtained for each observation as the ratio between the derivative of the estimated aggregate production function in relation to capital per effective labor and the average product of capital. Then, the local regression is employed to estimate the curve. Figure 11 presents the local regression plot of the elasticity of output with respect to capital. There are two segments in the fit. The first segment corresponds to observations of the capital per effective labor ratio between zero and \$ 2500 1996 PPP per effective labor with the elasticity of output with respect to capital increasing from 0.32 to 0.8. The second one corresponds to observations of the capital per effective labor ratio above \$ 2500 1996 PPP with the elasticity of output with respect to capital oscillating around 0.78. The result of the second segment is consistent with most of the empirical literature (ROMER, 1987; MANKIW, ROMER, and WEIL, 1992; WOLFF, 1991). There was not tendency for the estimated elasticity of capital to approximate to one. This result supports the Solow-Swan model.

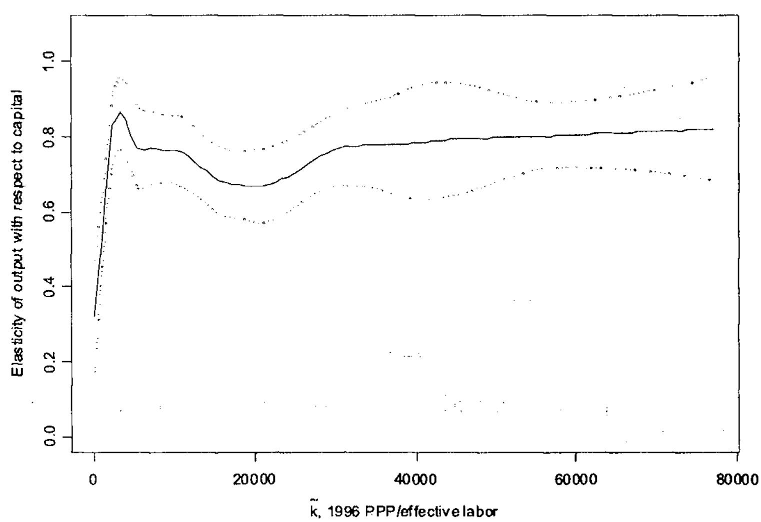

Figure 11. The local regression plot for the elasticity of output with respect to capital (HESTON, SUMMERS and ATEN, 2002, and our estimation of the net standardized fixed capital stock. Local regression parameters: bandwidth = 0.45, degree = 1, number of observations = 108).

There are three important features in this pattern of the capital contribution to output. First, there is the possibility of divergence in the growth rate of output per effective labor between poor and other countries in the Solow-Swan growth model. The value of $ 2500 1996 PPP per effective labor is just above the first quartile of observations for the capital per effective labor ratio in 1985. As it was stated above, the growth rate of output per effective labor can be written as $\hat{y} = \alpha\hat{k}$. Thus, if the typical elasticity of output with respect of capital for the countries in the first quartile is 0.5 and 0.78 for other countries, then the growth rate of capital per effective labor ratio for the countries in the first quartile must be 56 percent higher than in other countries to achieve convergence in output per effective labor.

Second, accumulation of capital has an important role in the expansion of output. The elasticity of output per effective labor with respect to saving, $\frac{d\tilde{y}^*}{ds}\frac{s}{\tilde{y}^*}=\frac{\alpha^*}{1-\alpha^*}$, is equal to 3.5 for the estimated α^* of 0.78. This result is consistent with our previous analysis of the effects of a change in investment over the steady state level of output per effective labor.

Third, the estimated α^* is compatible with previously estimated speeds of convergence. As pointed out by Romer (1994) and Barro and Sala-i-Martin (1995) for elasticity of output with respect to capital of the magnitude estimated in the second segment of Figure above the Solow-Swan growth model is consistent with an estimated speed of convergence of 2 percent per year. The estimated speed of convergence, $\beta = (1 - \alpha)(n + g + d)$, for countries with high capital per effective labor ratio is 2.15 percent per year.

Overall, the results on the elasticity of output with respect to capital are consistent with the Solow-Swan growth model. However, these results bring some embarrassment to the neoclassical theory of distribution, because the measured elasticity of output, 0.78, is so much higher than the typical capital share in developed countries, between 0.3 and 0.4. The estimated contribution of capital to output is twice of the observed profit share. Capital appears to be paid less than its marginal productivity to output. Mankiw, Romer, and Weil (1992) solve this problem with the addition of human capital in the Solow-Swan model in order that tangible and intangible capital have an output share of 0.66.

4 The "real" marginal returns to capital

In the Solow-Swan model the marginal product of capital declines to zero as the capital per effective labor ratio increases, while in the Jones and Manuelli (1990) a lower bound is assumed for profitability.

When this lower bound is reached the aggregate production function becomes linear and the marginal productivity of capital constant. Hence, the Solow-Swan growth model and Jones and Manuelli (1990) can be tested visually by looking at the behavior of the gross profit rate. It is equal to the marginal product of capital whether income is distributed according to the marginal productivity of the factors of production.

Figure 12 plots the local regression estimate for the gross profit rate. It was computed as the capital share multiplied by the capital productivity. The fit presents a negative correlation between gross profit rate and capital per effective labor ratio. However, the confidence intervals show that the hypothesis of a constant gross profit rate can be accepted at five percent for the capital per effective labor ratio above $ 45000 1996 PPP. Again, this result is inconclusive between Solow-Swan and Jones and Manuelli (1990) models.

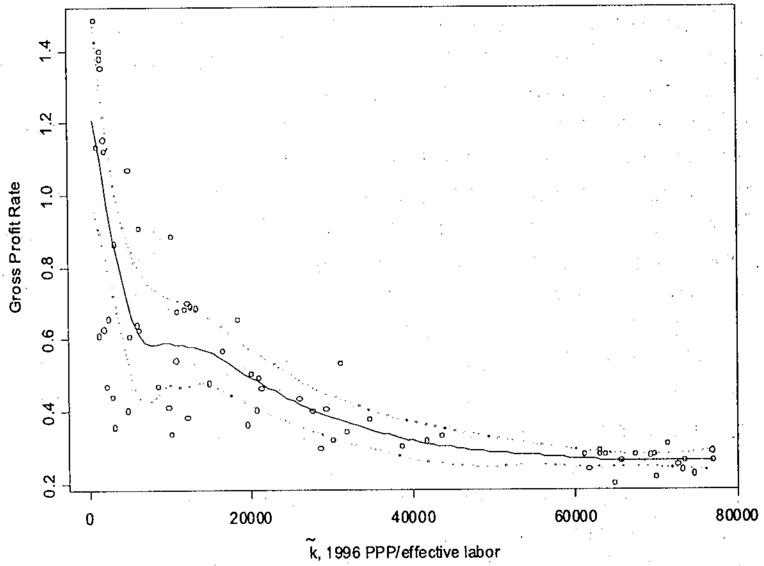

Figure 12. The local regression plot of the "real" marginal product of capital (HESTON, SUMMERS and ATEN, 2002, and our estimation of the net standardized fixed capital stock. Local regression parameters: bandwidth = 0.52, degree = 2, number of observations = 70).

5 Conclusion

The convergence controversy and related questions have occupied a leading role on the empirical debate of economic growth since the middle 1980s. The absence of convergence has been considered as favoring the endogenous model while the presence of convergence has been regarded as supportive of the Solow-Swan model. In the present paper we investigate some questions related to the process of convergence using a methodological procedure based on the non-parametric method called local regression. It allows us to plot the full representation of the Solow-Swan growth model and the elasticity of output with respect to capital. This methodology permits us to distinguish not only between the endogenous and the Solow-Swan model but also among the three groups of the endogenous growth models.

The local regression fit of the aggregate production function shows the presence of diminishing returns to capital, in particular at low levels of capital per effective labor ratio. It contradicts the endogenous models with constant and increasing returns to capital. The local regression plots of the phase diagram and the full representation of the Solow-Swan growth model did not give a clear answer between this model and Jones and Manuelli (1990).

The results on the capital contribution to output support the Solow-Swan growth model. The capital contribution to output is between 0.32 and 0.8 for countries with a capital per effective labor ratio between 0 and $ 2500 1996 PPP per effective labor ratio and it oscillates around 0.78 for countries with capital per effective labor above $ 2500 1996 PPP. Other three interesting results are the possibility of rich countries present higher growth rates in output per effective labor than poor countries within the Solow-Swan model, that the speed of convergence is in the order of 2.15 percent, and the estimated output elasticity with respect to savings is 3.5 percent. The evidence on the returns to capital indicated that the gross profit rate tends to decline as the capital per effective rate increases.

Altogether the results tend to support the exogenous growth model. It is consistent with a large body of the empirical literature on economic growth. Baumol (1986), Wolff (1991), Mankiw, Romer, and Weil (1992), Islam (1995), Evans and Karras (1996), and Binder and Pesaren (1999) obtained the same result. However, it is important to emphasize that the Jones and Manuelli (1990) model was supported for a series of statistical tests. It indicates that the endogenous growth model can be a close approximation for the countries with high capital per effective labor ratio, the leading countries in technology development.

References

AGHION, P.; HOWIT P. A model of growth through creative destruction. *Econometrica* 60, 385-406, 1992.

BARRO, R.; SALA-i-MARTIN, X. Convergence. *Journal of Political Economy*, 100, 223-51, 1992.

BAUMOL, W. Productivity Growth, Convergence and Welfare: What the Long Run Data Show? *American Economic Review*, 76, 1072-85, 1986.

BINDER, M.; PESARAN, M. H. *Stochastic Growth Models and Their Econometric Implications*. Cambridge University, unpublished, 1999.

CLEVELAND, W.; LOADER, C. Smoothing by Local Regression: Principles and Methods. In: HÄRDLE, W.; SCHIMEK, M. G. (eds.). *Statistical and Computational Aspects of Smoothing*. Heidelberg: Physica-Verlag, 1996.

EVANS, P.; KARRAS, G. Convergence Revisited. *Journal of Monetary Economics*, 37, 249-65, 1996.

GROSSMAN, G.; HELPMAN, E. *Innovation and Growth in the Global Economy*. Cambridge, MA: MIT Press, 1991.

HESTON, A.; SUMMERS, R.; ATEN, B. *Penn World Table 6.1*. Center for International Comparisons at the University of Pennsylvania, 2002.

ISLAM, N. Growth Empirics: a panel approach. *Quartely Journal of Economics* 110, 1127-70, 1995.

JONES, L.; MANUELLI, R. A Convex Model of Equilibrium Growth: Theory and Policy Implications. *Journal of Political Economy* 98, 1008-1038, 1990.

KING, R.; REBELO, S. Public Policy and Economic Growth: Developing Neoclassical Implications. *Journal of Political Economy* 98, 126-150, 1990.

KURZ, H.; NERI, S. Theories of "endogenous" growth in historical perspective. In: TEIXEIRA, J. (ed.). *Money, Growth, Distribution and Structural Change: Contemporaneous Analysis*. Brasilia: University of Brasilia Press, 1997.

LOADER, C. *Local Regression and Likelihood*. New York: Springer, 1999.

LONG, B. de; SUMMERS, L. Equipment Investment and Economic Growth. *Quarterly Journal of Economics*, 106, 445-502, 1991.

LUCAS, R. On the Mechanics of Economic Development. *Journal of Monetary Economics*, 22, 3-42, 1988.

MANKIW, N.; ROMER, D.; WEIL, D. A Contribution to the Empirics of Economic Growth. *Quarterly Journal of Economics*, 107, 407-37, 1992.

REBELO, S. Long-Run Policy Analysis and Long-Run Growth. *Journal of Political Economy* 99, 500-521, 1991.

ROMER, P. Increasing Returns and Long-Run Growth. *Journal of Political Economy*, 94, 1002-37, 1986.

ROMER, P. Endogenous Technological Change. *Journal of Political Economy*, 98, S71-S102, 1990.

ROMER, P. The Origins of Endogenous Growth. *Journal of Economic Perspectives*, 8, 3-22, 1994.

SOLOW, R. A Contribution to the Theory of Economic Growth. *Quarterly Journal of Economics* 70, 65-94, 1956.

WOLFF, E. Capital Formation and Productivity Convergence over the Long Term. *American Economic Review*, 81, 565-79, 1994.

YOUNG, A. A Tale of Two Cities: Factor Accumulation and Technical Change in Hong Kong and Singapure. In: BLANCHARD, O.; FISCHER, S. (eds.). *NBER Macroeconomics Annual 1992*. Cambridge: MIT Press, 1992.*Economy*, 94, 1002-37.

Appendix 1: Data source and methodology

This appendix presents a brief description of the methodology used to calculate the variables employed in the paper. The basic data source is the Penn World Table, Mark 6.0 (PWT v. 6.0). For the list of variables and the exposition of the PWT methodology, see Heston, Summers and Aten (2000). In the paper was employed a sample with 108 countries for 1985.

Real Gross Domestic Product utilized is the Chain Index expressed in 1996 purchasing power parity (PPP) (series RGDPCH). The output per effective labor, $\tilde{x}$ is measured as the real GDP per worker. It is presented in the PWT v. 6.0 (series RGDPW). The number of worker is the ratio between RGDPCH and RGDPW. The growth rate of the number of worker, n, was computed as $(\ln(n_t) - \ln(n_{t-5}))/5$. The saving rate, s, is the investment share in GDP (series I).

Capital per effective labor ratio, $\tilde{k}$, is the ratio between our net standardized fixed capital stock and the number of workers. Our net standardized fixed capital stock is obtained by the Perpetual Inventory Method (PIM) using the investment series computed from the variable real investment share of GDP presented in the PWT v. 6.0 The PIM procedure follows Hulton and Wycoff (1981). The depreciation takes a geometric form. Hulton and Wycoff (1981, p. 94) calculated the rate of depreciation (d) by d = R/T where R is the factor that defines the degree of declining balance due to depreciation, and T is the average asset life. The R employed for us is 1.05. The asset life considered was 14 years; hence the depreciation rate was 7.5 percent. The net capital stock was computed using the expression $K_T = \sum_{i=t}^{T} (1 - 0{,}075)^{(T-i)} Ii,\ i = t, \ldots, T$, where I is the investment series calculated from the variables real investment share of GDP, real GDP per capita in constant dollars (chain index), and population in the PWT v. 6.0.

Capital share was computed as π, where $1 - \pi$ is the wage share calculated as the employee compensation in the GDP. It is the only variable that is not obtained from the PWT, v. 6.0. The computation of the wage share for 70 countries in our sample is based on the following sources: United Nations. 1982, Yearbook of National Accounts Statistics 1980: International Tables, vol. II, New York; United Nations, 1989. National Accounts Statistics: Analysis of Main Aggregates, 1986, New York; and United Nations, 1994. National Accounts Statistics: Main Aggregates and Detailed Tables, 1992. New York.

Appendix 2: Local regression

Local regression is a non-parametric method of fitting data developed by Cleveland and Loader (1996) and Loader (1999). Local regression is implemented in Locfit, a software package developed for S or S-Plus. It can be obtained at http://cm.bell-labs.com/stat/project/locfit.

Local regression employs a combination of smoothing with weighted least squares. The procedure for computing one smoothed point $(x_i, \breve{y}_i)$ is the following. First, a segment of the scatter plot is defined. It is not centered in the point of interest (x_i, y_i) only for the points at far left and far right of the scatter plot. The bandwidth k must be chosen. Following Loader (1999, p. 20), we employ the nearest neighbor bandwidth κ, with $0 < \kappa \leq 1$. It indicates the proportion of observations that are considered in the computation of the smoothed point. It controls the smoothness of the fit. Generalized cross validation and Akaike's criterion were used in the bandwidth definition. Second, the neighborhood weights for all points within the segment are defined. The features of the weighted function are: 1) the point of interest (x_i, y_i) has the largest weight; 2) the weighted function is symmetrical and decreases as x moves away from x_i; 3) the weighted function is equal to zero in the boundaries of the segment. The tricube weight function was employed in the estimates. Third, a line is fitted using weighted least squares for the defined segment of the scatter plot. The fitted line is a *d*th degree polynomial fit. We followed the suggestions by Cleveland and Loader (1996) in the definition of the degree of fit in the estimates. The value of the fitted line at x_i defines $\breve{y}_i$. This procedure is repeated for all points of interest in the scatter plot. Then, the points of interest are linked, forming the fitted curve.