



Epistemic Closure and Epistemological Optimism

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Received: 12 September 2019 / Revised: 20 February 2020 / Accepted: 11 March 2020 /
Published online: 28 May 2020
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Abstract

Half a century later, a Dretskean stance on epistemic closure remains a minority view. Why? Mainly because critics have successfully poked holes in the epistemologies on which closure fails. However, none of the familiar pro-closure moves works against the counterexamples on display here. It is argued that these counterexamples pose the following dilemma: either accept that epistemic closure principles are false, and steal the thunder from those who attack classical logic on the basis of similarly problematic cases—specifically, relevance logicians and like-minded philosophers—or stick with closure and surrender to relevantist claims of failure in truth-preservation aimed at classical rules of inference (by Edwin Mares, Stephen Read, et al.). Classicist closure advocates find the promise of a way out of the dilemma in the works of Roy Sorensen and John Hawthorne. The paper argues against their pro-closure move and renews Robert Audi’s call for a theory of closure-failure.

Keywords Epistemic closure · Inferential knowledge · Skepticism · Epistemology of reasoning · Relevance logic · Moore’s paradox

Fifty years ago, the epistemology of reasoning gained a monstrous problem. That problem is still with us, scarier than it has ever been. Since then, it has, *inter alia*, become the problem of how one of Fred Dretske’s (1970, 2014) most insightful observations has been so badly mistreated by so many philosophers for so long. As we shall see, the mistreatment began at home, so to speak, with Dretske’s own mishandling of his groundbreaking observation about how deduction, as a source of

Versions of this paper were presented as invited talks at two conferences, in Germany, at The Epistemology of Reasoning conference, hosted by CONCEPT (Cologne Center for Contemporary Epistemology and the Kantian Tradition), and in Brazil, at the 11th International Principia Symposium, hosted by UFSC.

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inferential knowledge, is turned problematic by cases of apparent *epistemic-closure failure*, that is, by the apparent fact that valid deductive reasoning does not seem infallibly to transmit epistemic status from premises to conclusion.¹ As we shall see, progress in the matter calls for more caution than we have found in the literature on Dretske's problem. So, that's where I begin, with a couple of crucial distinctions without which we'd slip back into what Jonathan Kvanvig (2005) once called 'the closure mess': the misleading claims about which principles are under threat when authors influenced by Dretske bring up purported counterexamples to epistemic closure. Once the conceptual fog has been lifted just high enough for us to forge ahead, an assortment of counterexamples to closure will be paraded before your eyes, and we will then look at the most impressive pro-closure view that I have found in the literature on the issue. We will eventually find that the charms of this pro-closure move are skin-deep.

1 Closure and Skepticism

We have many reasons to be grateful to Dretske for the lessons he gave us, but, here, the relevant one is, by itself, a feat for the ages: the way he put epistemic closure principles under suspicion. He set out to question the idea that, as he put it, 'the epistemic worth of a proposition is hereditary under entailment' (1970, p. 34).² He then masterfully made the case that it is not invariably thus hereditary by giving us cases where transmission of epistemic status seems to fail in valid (deductive) reasoning. And, with that, he changed the epistemology of reasoning forever, even by the lights of those who have made a career out of warning against the Dretskean anti-closure stance. And, yet, all gratitude notwithstanding, I must demur at Dretske's way with the idea of closure failure. Why? Because, he occasionally does go very wrong in his handling of his problem, and I believe we're still fighting to keep all the good he did to the epistemology of reasoning from being overshadowed by the pernicious influence of what he did wrong. What am I talking about? About the two distinctions I should make before we look at the challenging cases.

The first distinction requires identifying the fundamental closure principle that Dretske put in jeopardy but failed to identify as clearly as it could have been identified. (He only hints at it.) It has become routine, in discussions of this issue, for writers to note that knowledge is not closed under entailment. It is not closed even under Dretske's *elitist* restriction to *known entailment*, since, elitism aside, one's believing the entailed propositions is the hazardous condition here. So, it is

¹ I regret not having the space to discuss the distinction between closure and transmission as proposed in the work of Crispin Wright. See Wright (2002) for a sample of the proposal. For context, see Moretti and Piazza (2018). Suffice it to say that, like many, I see what Wright calls 'transmission' as being what matters in discussions of epistemic closure.

² I should note, in passing, that I don't think propositions are bearers of *epistemic* properties. Strictly speaking, by itself, a proposition is not the kind of thing that can have 'epistemic worth' (none other than its being true). So, I believe we can charitably assume that the reference to epistemic properties of propositions—Dretske's talk of their 'epistemic worth'—can be understood as a reference to the epistemic status (or 'worth') of the relevant *propositional attitudes* of which those propositions are the content.

not too long before we are offered *pseudoclosure* principles involving the concepts of knowledge and entailment, such as the following very popular (elitist) principle: ‘If you know that p , and know that p entails q , then you are *in a position to know* that q .’³ Good luck understanding what is meant by one’s ‘being in a position to know’! Obscurity is, indeed, avoided when experts shift the focus from entailment to *valid deduction*, as in the following principle: ‘If you know that p and validly deduce that q on the basis of your knowledge that p , then you know that q .’⁴ But this is only an evasive maneuver. The appeal of the principle concerning one’s being *in a position to know* still awaits clarification. What is it in knowledge that is supposed to be ‘hereditary under entailment’, or hereditary under known entailment, or hereditary under known entailment with believed entailed propositions? No mystery here. The answer has been in the literature for decades now. It’s captured by the only principle we have known according to which epistemic worth is ‘hereditary under entailment’, a principle of *justification-transmission* by entailment (or ‘JC’, shorthand for ‘justification-closure’ henceforth):

(JC) If S is (*ultima facie*) justified in believing (i.e., *entitled* to believe) that p , and p entails q , then S is (*ultima facie*) justified in believing that q .

There is quite a bit to be said about why many find it optional to discuss JC when the epistemic closure issue is examined in the literature. But I must move on.⁵ So, here, I’ll simply note that it is exclusively to threats to JC that I will turn in what follows. Still, another, more exciting distinction must be made before we turn to what I take to be genuine counterexamples to JC.

When Dretske raised his seminal challenge to epistemic closure, he put it in the context of our collective struggle against (so-called) Cartesian skepticism. But, here, things go very wrong. Questioning epistemic closure principles was a brilliant idea;

³ In what follows, we can safely ignore the issues that are specific to the debate over multi-vs-single-premise closure. The case against multi-premise closure is extremely divisive, it’s one whose defining characteristics are inessential to the discussion offered here. See Pollock (1983) for guidance in the matter.

⁴ The most popular formulation of this principle—one that has garnered approval from authors such as Hawke (2014), Luper (2016), Pritchard (2016), Sharon and Spectre (2017a), and Schechter (2013), among others—was put forward by Hawthorne (2004). Here it is, with its innovative clause italicized by me: ‘Necessarily, if S knows p , competently deduces q [from p], and thereby comes to believe q , *while retaining knowledge of p throughout* [the deduction], then S knows q .’ Why do we need the italicized clause? Hawthorne suggests that, while the inferential process takes place, the premise-belief might conceivably lose its status as knowledge. But would there be the temptation to think of knowledge-closure failure in a case where the premise has lost its status as knowledge before the concluding belief is formed? (Would you be tempted to ask if x has *transmitted* to y what you believe that x didn’t have when the transmission was supposed to take place?) If not, the italicized clause is confusing, for raising the issue of a non-existent temptation. I submit that it is, indeed, confusing and may safely be ignored. When focusing on knowledge closure, we tacitly dismiss the case where the premise no longer has the epistemic goods to transmit when the conclusion is reached. (A related worry—possibly this very same worry in different verbal guise—is put forward by Alspector-Kelly (2019, pp. 9–11), who also finds that clause objectionable.) It should be noted that Hawthorne reaffirms his commitment to the confusing clause in Fraser and Hawthorne (2015), which is surprising, given that he claims to be suspicious of ‘the ideology of defeat’. But the motivation for the confusing clause comes from the idea that knowledge can be lost to misleading evidence (Hawthorne 2004, p. 33), and that idea is central to ‘the ideology of defeat’.

⁵ I tackle some of these issues in de Almeida (2019).

doing so as part of an anti-skepticism strategy, a very bad one. Let's see why. (But, here, I'm afraid I must be too quick for comfort. We have bigger fish to fry.)

Recall that, for Dretske and for many of us, the relevant skeptical argument is often aptly framed as a *modus tollens*. Here it is in Dretskean guise, as an episode of reasoning that a reflective zoo goer might find herself performing (or undergoing):

Zebras: While standing before the zebra-looking animals in a pen marked 'Zebras' at the local zoo, Zoo Goer is struck by a terrifying thought: 'If I'm justified in believing that those animals are zebras,' she notes, 'then—given that their being zebras entails that they're not cleverly disguised mules—I'm justified in believing that those animals are not zebra-looking mules. But, wait a minute! I'm not justified in believing that they are not zebra-looking mules—since, clearly, if they were zebra-looking mules, I'd still think they were zebras. So, as my epistemology teacher would put it, by *modus tollens*, I must admit that I'm not justified in believing that the animals are zebras—and that's in spite of the zebra looks and the 'Zebra' sign and the presumed seriousness of the zoo authorities!'

That, as you know, is how Dretske would have us think about the case, with 'justification', here, replacing his preferred term, 'knowledge', and all the worry remaining intact, not an iota of the issue lost in the replacement.

But here's the crucial question for us at this juncture: Is it JC that Dretske should have identified as the problematic element in Zoo Goer's justification for the conditional in her inference? Or is it the following principle, what became known as 'the underdetermination principle' (or 'UP')?

(UP) Body of evidence *e* (prima facie) justifies a given agent in believing that *p* (that is, provides epistemic prima facie entitlement for the agent to believe that *p*) only if *e* (prima facie) justifies the agent in believing the negations of any propositions that are incompatible with *p*.⁶

Notice that the Dretskean worry is that Zoo Goer's body of evidence underdetermines the rational choice between the proposition that those animals are zebras and the (incompatible) proposition that those animals are zebra-looking mules.⁷ The skeptical pressure on Dretske from *Zebras* is a clear case of Zoo

⁶ While retaining the essence of UP as it appears in the works of Brueckner (1994, 2005, 2010), or Vogel (1999, 2004), or Pritchard (2005, 2016), or Cohen (1998), among others, my way of expressing it differs from theirs. I don't have the space to compare formulations here.

⁷ Sharon and Spectre (2017a, p. 1009; 2017b, p. 1084) emphasize the fact that, depending on the available evidence, if *e* is good evidence for Zoo Goer to believe that she sees zebras, *e* may also be good evidence for her to *disbelieve* that the animals are not zebra-looking mules, since the evidence in question, say, zebra looks, may also be *some evidence* for the belief that the animals are zebra-looking mules. But I understand that, by UP, the skeptic means that *e* is *no evidence* for either of the competing claims ('those are zebras' and 'those are zebra-looking mules'). By contrast, at this juncture in their discussion of Dretskean cases, while concerned with the phenomenon of underdetermination, Sharon & Spectre (2017a, pp. 1010–12) part company with the literature that discusses the *skeptical* UP by deploying a different concept of underdetermination, one on which both competing claims *are equally supported* by *e* in such cases (even if rational belief remains impossible). If I'm not mistaken, the concept that matters to them is the one discussed, for instance, in Newton-Smith (1981, pp. 40–3).

Goer's having evidence that fails to satisfy UP. His concession to the skeptic is in acknowledging our tacit respect for UP, without distinguishing the principle from JC. (He goes so far as to accept the falsehood of the consequent in Zoo Goer's conditional premise, but, as we know, he then pivots on an explanation of why we should resist the conclusion delivered by *modus tollens*, thus opposing the skeptic by rejecting the epistemic principle on which the conditional is based.) And his failure to distinguish between JC and UP perniciously reverberated in the literature influenced by his groundbreaking JC-related work.

This is not where I can discuss the importance of distinguishing between JC and UP in any great detail. (Again, bigger fish awaits.) But it is where I should take the opportunity to draw your attention to the fact that UP is clearly logically weaker than JC. According to UP, a necessary condition of one's having evidential support for a given belief that p is that the evidence for that belief should also support belief in the negations of every proposition which is incompatible with p (whatever the relevant modality for incompatibility). But the class of the negations of propositions incompatible with p is a proper subclass of the logical consequences of p . By contrast, JC calls for the transmission of epistemic support (whatever may account for epistemic support in your epistemology) to every logical consequence of p . If this is right, UP-failure entails JC-failure, but not conversely.⁸

This matters to what follows because I will consider a number of cases of apparent JC-failure none of which counts as a counterexample to UP. And this brings us to my motivation for claiming that Dretske's proposed refutation of JC (while ostensibly speaking of knowledge-closure) is misleading with regard to skepticism. He aimed at promoting an anti-skepticism epistemology by giving us reasons to believe that the conditional in the skeptical argument was based on an unsound principle. But he had the wrong target in mind, since the skeptic, while armed with the weaker UP, can safely shrug her shoulders to *Zebras*-type counterexamples to JC. So, while cases such as *Zebras* may well be seen as a valuable anti-JC weapon, the whole connection with skepticism is a very unfortunate, but historically very significant, distraction. No anti-JC epistemology is, *eo ipso*, an anti-skepticism epistemology

2 Closure and Paradoxical Belief

Most philosophers who reflect on the closure issue are tempted to find the falsehood of JC incredible. Why? Maybe because truth-conduciveness is, since time immemorial, an essential feature of our pre-theoretical notion of reasonableness (or 'justification') in reasoning of any kind, and reasonableness in deductive reasoning is perceived as parasitic on truth-preservation, as the latter is regarded as evidence for the former.⁹ We find this way of speaking even in philosophical

⁸ A defense of this claim, by a different route, can be found in Pritchard (2005).

⁹ Undoubtedly, the intuitive requirement of truth-conduciveness for justification can come under fire from prima facie counterintuitive considerations of a skeptical nature, considerations which have proven hard to resist under reflection, as in Cohen's 'new evil demon' scenarios. See Cohen (1984), and the literature on it, for instance, Goldman (1988) and Sosa (1991).

discussion of an abstract epistemic principle such as JC. For instance, at a certain point in his exchange with Robert Audi, Richard Feldman (1995, pp. 493–4) claims that, as a JC denier, what Audi recommends is for you ‘to refuse to accept what you know to be the consequences of your beliefs,’ the kind of thing ‘we routinely counsel our first-year students not to do.’ (Notice the generalization in Feldman’s claim.) In like manner, while opposing (what was in effect) the Dretskean case against JC, John Hawthorne (2014, p. 46) suggests that the JC denier ‘resembles perfectly Lewis Carroll’s Tortoise, that familiar object of ridicule who was perfectly willing to accept the premises of a *modus ponens* argument but was unwilling to accept the conclusion.’ (But wait! Wasn’t Dretske at pains to explain why we may, on occasion, feel compelled not to reason classically without our having good-enough reason to claim invalidity? How ridiculous is that in view of skeptical scenarios such as *Zebras*?) Echoing these JC defenders, Peter Baumann (2011, pp. 600–01) suggests that a JC denier would oppose what ‘seems very hard to deny,’ to wit, ‘that we can acquire new knowledge by making inferences from what we already know. Denying closure (or transmission),’ Baumann warns, ‘seems to jeopardize the very idea of inferential knowledge.’ (But wait again! How do we soundly get from closure-denial to a threat against ‘the very idea of inferential knowledge’?)

All the doom and gloom and ridicule come from the idea that truth and justification are inseparable in deductive reasoning. If a case of reasoning exemplifies a pattern that we perceive as truth-preserving and delivers a conclusion that we take to be true from premises that seem (*ultima facie*) justified to us, then nothing else is required for us to be (*ultima facie*) justified in believing the conclusion. That’s why, I submit, we are being warned against confusing our first-year students. And there is something of undeniable *prima facie* appeal to this idea: if we reason from beliefs that we take to be justified to a conclusion we take to be false, we readily suspect either the justification we have for our premises or the validity of our inference (if it seemed valid to begin with). More generally, we naturally think that everything that suffices for us to regard a proposition as being true is sufficient to render us justified in believing the proposition. But, as it turns out, this line of thought is subtly misleading. Sometimes, truth and justification dramatically come apart, and what is clearly true to us is also perceived as rationally incredible, as the following cases show.

Here’s an example. Raymond Smullyan (1997, pp. 164–5) once thought of a philosophical prank that went as follows. Smullyan arrived early at the conference room where he would later speak. He made sure nobody saw him write a sentence on the blackboard, where it would remain until the beginning of his talk. The sentence was ‘You have no reason to believe this sentence.’ Once the audience was in the room, he asked a nine-year old boy if the boy ‘believed the sentence’. The boy replied that he did, since he noticed that, indeed, he had no reason to hold the belief. Smullyan rejoiced. He thought the boy’s answer was brilliant. But wait! Is that the right answer? If the boy’s belief is true, then it is not justified—assuming, of course, that one’s holding a belief that the believer, herself, regards as one for which she has no reason counts as having an irrational belief. Here, we find a case in which evidence of truth does not suffice to make the true proposition one that is *ultima facie* justified for the believer.

Slightly modified, the Smullyan sentence can be used to construct a counterexample to JC. Here's how.

Smullyan: Ray has just met his (very reliable) epistemology teacher. The teacher says: 'Ray, go to room 13. There, you will find a true sentence on the blackboard.' By hypothesis, on the basis of his teacher's testimony, Ray has just acquired an *ultima facie* justified belief, namely, the one expressed by

(1) *there is a true sentence on the blackboard at room 13.*

Once at room 13, Ray looks at the board, sees the only sentence written on it, and acquires the (by hypothesis, *ultima facie* justified) belief expressed by

(2) *the sentence on the board at room 13 is 'Ray is not ultima facie justified in believing the proposition expressed by this (very same) sentence.'*

So, now, Ray has a duo of (*ultima facie*) justified beliefs the contents of which entail a proposition that he is not (*ultima facie*) justified in believing, to wit,

(3) *Ray is not ultima facie justified in believing the proposition expressed by this (very same) sentence.*

There should be no doubt that Ray is *ultima facie* justified in holding the belief expressed by (2). He sees the board perfectly, he understands the only sentence on the board, and, by hypothesis, nothing else impinges on his justification for the belief. His belief in the proposition expressed by (1) should be acknowledged as equally unproblematic, being an ordinary case of testimonial justification. And notice that Ray is *prima facie justified* in believing (3), since he has excellent reason to believe it, namely, (1) & (2). But Ray can see that the belief in (3) is (ultimately) *unjustified if held by him*.¹⁰ So, it's unknowable to Ray, despite being contingent and knowable to his teacher.¹¹ There goes the claim that merely noticing that an inference is truth-preserving and proceeds from justified premises suffices for us to believe the conclusion with justification.

What has gone wrong? My answer is that things here go wrong only if you trust JC. But I expect resistance. So, I must ask: How does the counterexample fail? Certainly not just because it expresses a self-referring proposition. We have long known that self-reference is not, per se, problematic (Kneale and Kneale 1962, p. 228). Notice, also, that, if you delete the negation in (3), there will be no reason to deny that Ray's *prima facie* justification for believing (3) remains undefeated (that is, is *ultima facie*). Moreover, unlike the best-known paradoxes in the Liar family (The Contingent Liar, for instance), the operative concept in (3) is not semantic; it's epistemic—which suggests that seeking semantic anomaly for (3) is an uphill battle. (3) is unproblematically true.¹²

¹⁰ An explanation of why *prima facie* justification does not turn into *ultima facie* justification has been the *raison d'être* of the defeasibility theory of knowledge. (Its proposed resolution of the Gettier Problem is an application of that core idea regarding *prima/ultima facie* justification.) I cannot pursue the issue here. My present aim is simply that of offering a counterexample to JC.

¹¹ I assume, of course, that justification is necessary for knowledge, that is, that 'S knows that *p*, but she's not justified in believing it' is a piece of nonsense (or a necessary falsehood) that nobody should try to dignify.

¹² The fact that its truth makes it ultimately unjustified for Ray should lead us to the thought that (3) defeats Ray's justification for believing it.

So, why would our withholding assent to JC seem an unreasonable response to the case?¹³

In the case that follows, again, truth-preservation from justified beliefs fails to lead to justified belief, but, unlike the *Smullyan* case, this next case does not involve a self-referring proposition, nor does it involve a self-referring belief. The case assumes that (BD) *belief distributes over conjunction*, hardly a problematic assumption by most accounts. And it also appeals to the lemma according to which, (L) *if, at the same time, S both believes that p and believes that ~p, S must hold an unjustified belief*. But I can't expect you to take L for granted. So, I should offer you a proof of the lemma. I submit that there is a simple proof of L from the following core ideas of the defeasibility theory of knowledge: (D1) *any justifier of a belief that p (at a given time t) is an overrider of any prima facie justification one might have to believe that ~p (at t)*—or, in Pollockian terminology (Pollock 1986, p. 38), any justifier of a belief that p is a *rebutting defeater* of any justification for a belief that ~p; and (D2) *the defeating effect of a defeater can, itself, be defeated by a restorer (r) of an apparently defeated justification* (that is, there is such a thing as *pseudodefeat*). Thus, on D1 and D2, adding a reason to believe that p to S's belief system amounts to adding a rebutting defeater to any prima facie

¹³ The Smullyan case is originally from de Almeida (2012). A few years later, Fraser and Hawthorne (2015) put forward a case that closely resembles the Smullyan scenario, claiming that it is a threat to doxastic justification-closure. In both their case and mine, truth is incompatible with justification. Both are cases of epistemological appropriation of features of paradoxes in the Liar family that were discussed by two fourteenth-century philosophers, Jean Buridan and Albert of Saxony. (See Rescher 2001, pp. 193–213; Sorensen 2001, pp. 180–1, and 2003, pp. 210–11.) But their case is much more closely related to Buridan's Ninth Sophism (The Contingent Liar) than mine (a distinction they have failed to acknowledge). Here's how I schematically reconstruct their case. Consider the scenario in which an episode of reasoning by character B is preceded by justifying testimony by character A, to B, resulting in the following factors:

- (1) A's justified belief: *None of B's deductively justified beliefs at t is true;*
- (2) B's testimonially justified belief at $t-1$ ($< t$): *(j) None of my deductively justified beliefs at t is true;*
- (3) Assumption for *reductio*: B's only deductively justified belief at t, with false f, is that $j \vee f$.

And here's how I think you should proceed to derive the problem. (Their explanation of it is not entirely felicitous.) Assume bivalence (that is, the principle that every proposition is either true or false, but not both). Now, ask yourself if the belief that $j \vee f$ is true. If it is, then it's not deductively justified, since its truth must depend on a true j. But j is true iff $j \vee f$ is not both true and deductively justified. So, $j \vee f$ can't be both true and deductively justified. Now, can it be false and deductively justified? No. If it were, j would have to be false—in which case, there would have to be something believed by B at t that is both true and deductively justified. But, as we have seen, that's impossible. So, by bivalence, j cannot both be deductively justified and have a truth-value. So, while keeping bivalence, we must accept that $j \vee f$ is not deductively justified for B, and, by this route, justification-closure is now under the gun. But notice this trait of Fraser & Hawthorne's case: Since the justification for $j \vee f$ depends on a justified belief that j, what is to keep us from using the derived incompatibility between bivalence and justification as a *reductio* of the assumption that j is (ever) ultimately justified for B, prima facie impressions notwithstanding, thus preserving justification-closure? Apparently, the doubt is unavoidable, unless it is a live option for us to see the argument as a threat to either bivalence or the validity of Addition. And here is the kinship with Buridan's Ninth Sophism: j, as believed by B, does not initially look paradoxical, but, then, we find a context in which it makes trouble for bivalence. By contrast, the Smullyan case does not obviously expose classical logic to suspicion. I cannot, here, further explore the significance of the contrast. (I thank an anonymous referee for *Philosophia* for bringing the Fraser & Hawthorne paper to my attention.)

(propositional) justification S might have had to believe that $\sim p$, but the defeat may be illusory.¹⁴

If these assumptions from the defeasibility theory seem right to you, here's a simple proof of L from D1 and D2. Let ' eJp ' abbreviate 'piece of evidence e ultimately (propositionally) justifies agent S in believing that p .' And assume, for *reductio*, that, at any given time, the following conjunction is true: $eJp \ \& \ e^*J\sim p$. By D1, we'd be tempted to conclude that, while harboring both e and e^* in her total evidence set, S would be ultimately justified in believing neither p nor $\sim p$, since there would be a rebutting defeater for (the justifications of) each of the contradictories, namely, e and e^* . However, by D2, the problem now is: Can both p and $\sim p$ have their justifications restored? The answer is: no. Suppose that $(e\&e^*\&r)Jp$. In that case, by D1, there is now, in S's total evidence set, a new rebutting defeater of the justification for the belief that $\sim p$, namely, $e\&e^*\&r$. So, if there were a restorer, r^* , of the defeated justification for $\sim p$ by $e\&e^*\&r$, then, by D1, there would be a rebutting defeater for the justification of the belief that p , namely, $e\&e^*\&r\&r^*$, and so on indefinitely, with each restoring conjunction turning into a rebutting defeater of the restored justification of the previous round for either p or $\sim p$. So, by D1 and D2, the conjunction $eJp \ \& \ e^*J\sim p$ is a necessary falsehood.

With BD and L in hand, I can offer you another counterexample to JC. Consider the following scenario:

Moore: Suppose that, after years of careful observation, I've come to hold the justified belief that my accountant is extremely reliable when talking about my finances and about how I relate to them. On that basis, I believe that

(1) *everything my accountant says about me and my finances is true*,

which, assuming it true and non-Gettierized, seems to be an ordinary case of inductive knowledge. Today, I get a message from my accountant in which he scolds me for my persistently reckless way of burning cash. He concludes his message by saying 'you're broke, but you believe you're not.' I examine the message carefully, and, on that basis, become justified in believing that

(2) *my reliable accountant says that (both) I'm broke, but I believe I'm not*.

So, I validly infer, from (1) and (2), the proposition belief in which might instill some sense in me and motivate responsible behavior, to wit,

(3) *I'm broke, but I believe I'm not*.

Can I be justified in believing (3)? Not if (3) is true. If it is true, its right-hand conjunct is true, in which case I believe that I'm not broke. But, by BD, I also (simultaneously) believe that

¹⁴ It is widely acknowledged that Klein's (1981) theory of 'misleading defeaters' is the best account we have of the phenomenon of pseudodefeat. There is relevant commentary on pseudodefeat in de Almeida and Fetz (2016). I should also note that the defeasibility theory imposes no constraint on the nature of justifiers. Although, in what follows, I speak of 'pieces of evidence' as justifiers, we may, instead, want to speak of belief-forming processes as justifiers. Going the reliabilist route is bound to get messy, as we know from Goldman's (1979) tentative talk of epistemic defeat as (on occasion) involving the *availability* of processes leading to the formation of beliefs that would be inconsistent with a given prima facie justified belief. But the problem of how epistemic irrationality is accounted for in purely externalist terms is not to be credited to the defeasibility theory.

I'm broke (that is, I believe the left-hand conjunct).¹⁵ But, by L, a true belief of (3) guarantees that my belief system harbors an ultimately unjustified belief. So, if all of my other beliefs are justified, the unjustified belief is a consequence of my believing (3). And it's obviously as absurd to deny that all of my other beliefs can be justified while (3) is believed as it is to deny that we can consistently stipulate that, in the *Moore* scenario, (1) and (2) are true. Since the argument is clearly valid, (3) is true. But, since the hypothesis of an ultimately justified belief of (3) (given BD and L) entails that the believer has an ultimately unjustified belief, we have a *reductio* of the hypothesis that (3) is ultimately justified. So, JC seems to fail here.¹⁶

And, again, I expect resistance. But how can it present itself? Maybe it will go like this: 'Wait a minute, mister closure-denier! (3) is a case of the commissive form of Moore's Paradox. It's paradoxical!' Yes, it is. But why is that a problem when I use it to make trouble for JC? I can't see an articulate answer coming my way. But I suspect that, whatever form that answer might exactly take, it will involve a double standard. If you are a JC loyalist, I'm giving you a new reason to regard the commissive form of Moore's Paradox as paradoxical—namely, the fact that it makes trouble for JC, and that's not something you were expecting. But, if, generally speaking, there is a problem with using a paradoxical proposition to make trouble for a philosophical principle, I wonder why people have no objection when, say, a Liar sentence is used to make trouble for the principle of bivalence. If there is no such double standard in play, the problem can't simply be in the fact that you already have *other reasons* to regard the Moorean proposition (3) as paradoxical, since that would be beside the point. You would still be expected to object to *this particular use* of the Moorean sentence and explain what's objectionable about *it*. You would still be looking at explaining how valid reasoning from true, justified beliefs leads to a paradoxical, true and unjustified, concluding belief.

3 Closure and Validity

In 1988, two influential philosophers, Robert Audi and Roy Sorensen, independently discussed a case according to which adherence to both classical logic and epistemic closure principles leads to a clear instance of dogmatism. Neither seemed aware of what the other was doing with the case. Audi focused on apparent closure failure and ignored the threat to classical rules of inference, apparently taking classical logic for granted.¹⁷ Sorensen ignored the threat to closure and focused on attacks to classical logic from the relevance logic camp (without explicit reference to relevance logic), apparently taking JC for granted, but speaking of inferential *knowledge*, not inferential *justification*, without assuming anything that might conflict with the traditional view of knowledge as implying justification.¹⁸ We owe it to Hawthorne (2004) to have

¹⁵ And, yes, by BD, I also believe that I believe that I'm not broke, but this higher-order belief is irrelevant to the argument.

¹⁶ An anonymous referee for *Philosophia* provided me with an excellent objection to an earlier argument from the Moore scenario. The present argument is in response to that objection.

¹⁷ To my mind, Audi (1991) persuasively defended his anti-closure case against criticism provided by Canary and Odegard (1989).

¹⁸ Sorensen acknowledges Ginet's (1980) discussion of the Kripke-Harman version of what became known as the 'dogmatism paradox'. But Sorensen's version of the problem is quite a bit more compelling than Kripke's recent version of his own original problem. For a discussion of Kripke's 2011 version of his problem, see Borges (2015).

noticed that Sorensen's defense of classical rules should be seen as a pro-closure move, and for claiming that its presumed success would dispose of Audi's anti-closure stance. As we look at the cases that motivated the three philosophers, we find the opportunity to understand that the threat posed to classical inference rules by 'relevantist' logicians is a case of tacit allegiance to JC—be they card-carrying relevance logicians, such as Edwin Mares (2004) and Stephen Read (1988), be they anti-classicist snipers who haven't endorsed any particular system of relevance logic, such as Robert Stalnaker (1975), Vann McGee (1985), Ernest Adams (1988), Jonathan Bennett (2003), or David Sanford (2003), among others.¹⁹ Looking at the broad range of cases that we find in both epistemology and the anti-classicist literature, it clearly seems that we are confronted with a choice: either let JC go the way of failed epistemological principles and ask yourself what is left of anti-classicist threats from the relevance camp, or stick with JC and ask yourself if you can deflect the purported counterexamples to classical validity (and classical semantics) put forward by relevantists.

The following case combines elements from the cases discussed by Audi (1988, 2011) and Sorensen (1988), without being completely faithful to either of their original cases.

Dogmatism: Roy is carefully working on his income tax forms. He adds a long column of figures and, on that basis, acquires the justified belief (in fact, the knowledge) that

(1) $253.75 + 43.20 + \dots + 17.42 = 756,000.31$ [a priori knowledge].

Soon afterwards, Roy notices that

(2) *if* $253.75 + 43.20 + \dots + 17.42 = 756,000.31$ *and somebody denies it, the person is wrong.* [a priori knowledge]²⁰

From this, he cannot help inferring that

(3) *if* $253.75 + 43.20 + \dots + 17.42 = 756,000.31$, *then, if somebody denies it, the person is wrong* [from 2, by Exportation].

Being so firmly based on a priori knowledge and classical logic, isn't it natural for him to conclude that

(4) *if somebody denies that* $253.75 + 43.20 + \dots + 17.42 = 756,000.31$, *the person is wrong* [from 1 and 3, by *modus ponens*]?

But, now, Roy is in trouble. Suppose a team of experienced auditors at the IRS, after reviewing his tax forms, concludes that

(5) *according to the team of IRS auditors, it's false that* $253.75 + 43.20 + \dots + 17.42 = 756,000.31$,

and they let him know about their conclusion. Should he believe, by Existential Generalization and *modus ponens* that a team of superior arithmeticians is wrong? 'Ah...'—you may object—'but, if the IRS team denies it, Roy loses his justification for believing (1), and, so, JC is not threatened!' But you would be missing the point. The point is: Can Roy reasonably believe (4), based on (1)-(3), if, as a matter of fact, nobody ever denies (1)?

¹⁹ I follow Burgess (2005) and use the generic label 'relevantism' for the kind of attack on classical rules of inference that we find in the works of the aforementioned authors.

²⁰ To simplify the deduction as much as possible, take 'the person is wrong' as short for 'whoever denies that [long sum] is wrong'.

There is no negative answer to that question that is not either a denial of JC, or a claim of invalidity aimed at either Exportation or *modus ponens*.

Sorensen noticed the similarity between the *Dogmatism* case and a 1985 case against *modus ponens* put forward by Vann McGee. Here's (a more explicit version of) the McGee case:

McGee: In the 1980 US presidential race, Republican Reagan was, according to reliable pollsters, way ahead of Democrat Carter. Based on the data provided by those reliable testifiers, Vann formed the justified belief that

(1) *Reagan will win, and he's a Republican* [by testimonial justification].

And that belief made it inevitable for Vann to believe that

(2) *A Republican will win* [from 1, by Existential Generalization].

But there was another Republican (and only that other one) in the race, Anderson, running as an independent, who was way behind both Reagan and Carter. So, again, based on reliable sources, Vann formed the belief that

(3) *if a Republican wins, it will be either Reagan or Anderson* [by testimonial justification].

Given his great logical acumen, Vann couldn't help noticing that,

(4) *if a Republican wins and the winner is not Reagan, then it's Anderson* [from 1, 2 and 3, by Conditional Proof, Simplification, *modus ponens*, and Disjunctive Syllogism].

From (4), by Exportation, Vann inferred that,

(5) *if a Republican will win, then, if Reagan is not the winner, it's Anderson*.

And that's how Vann was forced to confront the temptation to believe that,

(6) *if Reagan is not the winner, then it's Anderson* [from 2 and 5, by *modus ponens*].

But Vann knew better than to draw such a conclusion.

There is nothing in the above cases that would surprise relevance logicians. According to them, given the classical truth-conditions for both disjunction and the indicative 'if'—that is, the view that the indicative 'if' is a material conditional—there is an easy case to be made for the invalidity of Conditional Proof, Exportation, Disjunctive Syllogism, Or-to-if, Contraposition, Hypothetical Syllogism, Antecedent Strengthening, *modus tollens*, and *modus ponens*.²¹ Sorensen and Hawthorne, for their part, speak for most of us when they refuse to surrender any of the classical rules involved in the cases that motivate the relevantists. But it's Audi who speaks for me when he points to JC as the toxic element in the mix.

As you may have noticed, I have now put myself in a position to produce as many counterexamples to JC as you may want me to. I simply let relevance logicians take the lead and furnish me with instances of apparent non sequitur from the application of classical rules. Every case of apparent failure in truth-preservation that I have seen in

²¹ The best purported counterexamples to Hypothetical Syllogism and *modus tollens* I've seen are in Jackson (1987). MT is targeted in Adams (1988). The rule is defended in Sinnott-Armstrong et al. (1990). My favorite counterexample to Contraposition is in Sanford (2003, p. 227). Conditional Proof gives us some of the (improperly so-called) 'paradoxes of material implication'. I offer a case in de Almeida (2019). There are useful counterexamples to Antecedent Strengthening in both Jackson (1987) and Woods (1997). For a classicist explanation of why there seem to be counterexamples to Exportation, see Jackson (1987, pp. 127–31).

relevantist literature can also reasonably be regarded as a case of failure in justification-preservation in classical reasoning. When an argument that you regard as valid leads from seemingly true premises to a seemingly false conclusion, we naturally regard the premises as *bad reasons* for a belief in the conclusion. As our cases vividly show, all you then need for an eye-popping problem is a set-up according to which those premises seem justified. So, why should it be obvious that, as relevantists would have it, classical validity is the skunk in the cases? You won't find the answer to this crucial question in relevantist literature, since they all seem to take JC for granted.

I'll give you one last case before turning to classicist reaction from the JC camp. This is a case based on Hawthorne's Sorensenian discussion of how Addition might be used to make trouble for epistemic closure principles.²²

The mayor: Based on careful inquiry, young Dorothy has come to believe that (1) *the mayor is a model citizen* [by testimonial, perceptual, and inferential justification].

Given that she is a sharp classical reasoner, Dorothy notices that she has now put herself in a position to infer that

(2) *either the mayor is a model citizen or she is Tony Soprano's secret accomplice at City Hall* [from 1, by Addition].

And, according to Dorothy's epistemology teacher, whenever she reasons by Addition from what she's justified in believing, she must remain justified. But she notices that, if the disjunction is justified, there should be no objection to her believing that,

(3) *if the mayor is (for any reason) not a model citizen, she is Soprano's accomplice at City Hall* [from 2, by Or-to-if],

all of it without a shred of evidence of any Soprano involvement by the mayor! Stupefied by the thought, Dorothy is developing a headache. She's asking herself: 'Doesn't a justified belief in a disjunction require that each disjunct be a live epistemic possibility for the believer? Wouldn't normal people who knew about my admiration for the mayor reasonably judge me crazy if I believed the "Soprano disjunction"? Can I really *look* crazy and yet *be* rational?' It's a splitting headache now.

4 Closure and Epistemological Optimism

Closure defenders still have the masses on their side. There have been a number of pro-closure moves since Dretske made his epic case against closure, almost all of them designed to discredit epistemologies on which closure fails. But the debate involving contextualists and pro-closure invariantists has been fueled by parochial concerns in our struggle against skepticism. And it may already be clear to you that refuting anti-closure epistemologies is unlikely to silence the counterexamples we have seen. In any case, in what follows, I can only briefly review the most impressive pro-closure move I have found in the literature on these issues: the pro-closure use made by Hawthorne

²² In slightly different guise, the case is originally from de Almeida (2017).

(2004) of ideas developed by Sorensen (1988). It is impressive because it seems, at first glance, to be the only pro-closure move that smoothly applies to all of the anti-JC cases we have seen.²³

According to Sorensen, cases such as *Dogmatism* and *McGee* do look like impressive threats to *modus ponens*—and, more specifically, to Jackson’s (1979, 1987) view of the truth-conditions of the indicative ‘if’ as coinciding with those of ‘ \supset ’—unless we acknowledge the easy-to-miss fact that there is a species of *unassertable knowledge* that we may call ‘junk knowledge’, a phenomenon the acknowledgment of which, according to Sorensen, will take care of recalcitrant cases in Jackson’s view of the indicative conditional. Junk knowledge is the kind of knowledge that you cannot express without perpetrating a ‘conversational misdeed’ of an interesting kind. Examples of conversational misdeeds are easily collected in the philosophical literature. For instance, if I see you attracted to what I know are perfectly safe mushrooms, I shouldn’t assert that *I believe those mushrooms won’t make you terribly ill*, or that *if I didn’t buy the wrong kind again, those mushrooms are safe to eat*. Or consider: the fact that I know that conjunction is commutative and that the princess is a paragon of conservatism does not make it okay for me to assert that *the princess became pregnant and married her boyfriend*. It doesn’t much matter to Sorensen how the unassertability of these italicized sentences (or of the propositions they express) is explained (though he pays lip-service to Jackson’s ‘unassertibility’ view). What matters is that, in cases of the kind, there is the familiar temptation to confuse unassertability with falsehood, and, thus, the opportunity to miss the fact that the speaker expresses *unassertable knowledge*. Junk knowledge is supposed to be a kind of unassertable knowledge—specifically, the kind that one cannot use in classical reasoning to expand one’s stock of knowledge.

Notice how this is supposed to provide relief to classicists—and specifically to those classicists who also are JC-defenders, on Hawthorne’s account of the closure issue. Consider the *Dogmatism* case. There, according to Sorensen, your knowledge of

(4) *if somebody denies that $253.75 + 43.20 + \dots + 17.42 = 756,000.31$, the person is wrong.*

is junk knowledge. You cannot use the conditional to reason by *modus ponens* in the case. Upon getting a denial of your mathematical claim from a person whom you recognize as being at least an epistemic peer of yours in the matter, your justification for believing

(1) $253.75 + 43.20 + \dots + 17.42 = 756,000.31$.

will be weakened to the point where the belief ceases to be useful in the deduction. Instead of performing the dogmatic case of *modus ponens*, you should then lose your belief in (1), thus stopping the whole episode of reasoning. So, on a Sorensenian view of the matter, once we acknowledge the junk-like nature of that conditional premise, (4), neither JC nor the validity of *modus ponens* will be in the crosshairs.

Or consider our *Mayor* case. According to the approach favored by both Sorensen and Hawthorne, even though you know the disjunction expressed by (2), ‘either the mayor is a model citizen or she is Tony Soprano’s secret accomplice at City Hall’,

²³ In what follows, I build on discussion originally offered in de Almeida (2019).

based on your knowledge of the left-hand disjunct, if you were to find out that the mayor is a less-than-model citizen, for having, say, an unpaid parking ticket, you wouldn't reason by Disjunctive Syllogism and conclude that she works for Soprano. (2) is both known and inferentially useless; it's a case of disjunctive junk knowledge, according to them.

Similar considerations make it tempting to conclude that the junk-knowledge proposal might provide us with a respectable line of defense of JC for classicists in the other cases. But, alas, it does not survive scrutiny. I can give you three objections to the proposal.

First, knowledge-closure (or JC) cannot be protected by the junk-knowledge view of our counterexamples. By Hawthorne's and Sorensen's own admission, junk knowledge is a kind of second-class knowledge. That's the point of calling it 'junk'. First-class knowledge—or knowledge *simpliciter*—is the kind that does not suffer from this peculiar form of handicap: inferential worthlessness. So, if, for instance, your junk knowledge of a disjunction is based exclusively on your first-class knowledge of one of the disjuncts, as in the *Mayor* case, first-class knowledge is not inherited by the conclusion of your episode of Addition. If you know (*simpliciter*) that p and infer, on that basis alone, that $p \vee q$, you don't *know* the disjunction; you only *junk-know* it. Epistemic status is simply not inherited by valid deduction when the output belief is a case of junk knowledge (or 'junk justification'). As a pro-closure move, the junk-knowledge proposal is *not even in the ballpark*.

Second, even if we put Hawthorne's pro-closure hopes aside, problems arise for the junk-knowledge proposal regardless of application. Think of the *Mayor* case, for instance. The problem there is that the most natural explanation of what is wrong with your *asserting* the 'Soprano disjunction' is that you seem irrational in *believing* it. Here, conversational misdeed from sincere assertion is a sure-fire sign of madness. If you asserted the disjunction while admitting that you can only speak well of the mayor, your hearer would think you insane. And the best explanation for why you would be regarded as insane by any non-philosophical person who knew about your admiration for the mayor is that you *are* insane, if you believe the disjunction. Any discussion of *assertability*-conditions bearing on the case must focus on failure of *justification-preservation* in the inference. And the object of epistemic justification is a propositional attitude, not a speech act. If I'm not much mistaken, any story about how a distinction between assertability-conditions and truth-conditions might effectively protect classical logic will have to involve a theory of closure-failure as a key element. But then much of what we hear about *assertability* will turn out to be primarily about epistemic rationality from an *egocentric* point of view, and only derivatively about 'conversational misdeeds'. (But, yes, there is this respectable alternative: you may prefer to go relevantistic and claim that 'either...or...' is ambiguous, as Stephen Read 1988 has done when discussing analogous cases.)

Third, some apparently clear cases of ignorance turn out to be cases of junk knowledge on Sorensen's proposal. But these are cases where I suppose we're not the least bit inclined to think that there is *any* knowledge whatever in sight, including second-class, junk knowledge. For instance, according to Sorensen (1988, pp. 445–6), the Gettier case of Brown-in-Barcelona turns out to be a case of junk knowledge. But most of us would complain that the notion of knowledge is being grossly abused by the

junk-knowledge view. The Brown-in-Barcelona case is one where the ‘truth connection’ is completely accidental. Who among us would sanction a view of knowledge—*any* knowledge, regardless of class—where the truth connection is thus accidental? For another instance, think of our *Moore* case above. In order to view the conclusion of that valid argument, namely

(3) *I'm broke, but I believe I'm not,*

as a case of knowledge—*any* knowledge—you should demand a refutation of our argument (in §2) according to which a belief of (3) is either unjustified or false. Or think of the *Smullyan* case. There isn't a modicum of plausibility to the idea that, for Ray, the concluding belief is a case of something we'd call ‘knowledge’, since, in that case, his belief cannot be both true and justified.²⁴

5 Concluding Remarks

In 1995, under attack from Feldman, Audi (1995) stuck to his guns as he defended his view that (his version of) the *Dogmatism* case showed the need for a theory of closure-failure. A quarter of a century later, I'm echoing Audi's call for a theory of closure-failure and suggesting that the evidence remains every bit as hostile to closure principles—even more so now, I'd say—as it was when Dretske first wrote about it. Any epistemology of deductive reasoning that is based on (unrestricted) closure principles is, I submit, a case of simple-mindedness. It will remain vulnerable to counterexemplification. In the meantime, we are bombarded with disinformation and misleading claims by the pro-closure camp.²⁵ Recall Feldman's admonition to Audi, according to which the latter's anti-closure stance does a disservice to our first-year students. Diametrically opposed to Feldman, I've striven to suggest that it's high time our first-year students stopped being told that truth-preserving inferences are unproblematically justification-preserving.

In his *Routledge Companion to Epistemology* article on closure, Baumann (2011) offers a section entitled ‘Importance: Skepticism’, wherein he explains that the battle over epistemic closure principles is motivated by the dispute between skeptics, their invariantist pro-closure rivals, the Neo-Mooreans, the anti-closure Dretskean, or Nozickian (Nozick 1981), invariantists, and the pro-closure contextualists. There is no denying that, *as luck would have it*, our concern with closure came from that debate. But maybe the foregoing discussion has persuaded you that the motivation to seek a theory of closure-failure would still have been with us even if we had never paid any attention to skeptical arguments, or heard from philosophers called ‘Dretske’ or ‘Nozick’.

²⁴ Sharon and Spectre (2010) have offered what I think is the first extensive discussion of Sorensen's junk-knowledge proposal as applied to the dogmatism problem specifically. There is affinity between their appraisal of the proposal and my own. They, too, have found the notion of junk knowledge suspect. Maybe my objections to the proposal are what we need to close the case against it. (I thank an anonymous referee for *Philosophia* for bringing their work to my attention.)

²⁵ Some of the misleading claims have come from Luper's (2016) work on the issue. See de Almeida (2019).

The last decade has witnessed the arrival of a number of exciting hypotheses on both sides of the closure issue—by authors such as Marc Alspecter-Kelly (2019), Michael Blome-Tillmann (2015), Juan Comesaña (2014, 2017), Rachel Fraser and John Hawthorne (2015), Peter Hawke (2014), Assaf Sharon and Levi Spectre (2017a), Joshua Schechter (2013), Jonathan Vogel (2014), and Stephen Yablo (2017), among others. This is not the place where I undertake to review their works.²⁶ It's the place where I claim that it is far from clear how the new hypotheses they bring to the table may account for *all* of the cases set forth here, how they might be patched together, or tweaked, or supplemented, to give us either a complete theory of closure-failure or a complete theory of the illusion of closure-failure.

Acknowledgments For illuminating discussion, I'm grateful to my audiences at the conferences mentioned above, especially to Otávio Bueno, Sandy Goldberg, John Greco, Luis Rosa, and Paul Silva. I am most especially grateful to Robert Audi and to an anonymous referee for *Philosophia*, both of whom provided me with helpful comments on versions of the paper.

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²⁶ Their works all connect with important aspects of what we've seen here, but in multifarious ways, in ways that bring forth aspects of some of the problematic cases considered above. Some of their works are ostensibly aimed at theory-building (e.g., Yablo's); some are data-finding missions (e.g., Schechter's); and some pursue both objectives (e.g., Sharon & Spectre's).

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