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Improved Servomechanism Control Design - Nonswitching Case ⋆

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Abstract: This paper attempts to achieve time-optimal performance for servomechanisms via a practical feedback control law. An improvement over the traditional Proximate Time Optimal Servomechanism (PTOS) is proposed in order to eliminate the conservatism present in the controller. The original PTOS switches from the nonlinear approximation of Time Optimal Control (TOC) to a linear controller as the system approaches the reference point. This switching results in a compromise in performance inasmuch as the linear controller is not sufficiently aggressive to settle the system output with acceptable levels of overshoot. The proposed controller eliminates the necessity of the switching function by making use of an elaborate nonlinear control law. Simulations and experimental results show that the proposed design achieves levels of performance comparable to that of the theoretical TOC.

1. INTRODUCTION

Nonlinear controllers have frequently been applied to linear systems in order to achieve performance enhancements. This is particularly the case when it comes to fast tracking response of servomechanisms because the theoretical Time Optimal Control (TOC) itself is a nonlinear controller, Bryson and Ho, [1975]. In fact, fast tracking response is one of the most desirable performance criteria for servomechanism and, despite the vast amount of academic work in the area and the decades of years since the development of TOC, there still is a significant gap between the theoretical limits imposed by TOC and the performance achieved by practical controllers. Ideally, it would be preferable to implement TOC itself (also known as bang-bang), but it is well known that this controller is not implementable due to chattering caused by disturbances, measurement noise and model uncertainties, Khalil, [2002]. This paper proposes a control method that significantly closes the gap between the practical controllers and the time optimal one, while eliminating problems due to chattering.

Arguably, the most important work towards time optimal performance of servomechanisms is the Proximate Time Optimal Servomechanism (PTOS) proposed by Workman et al., [1987]. The general concept is to design a controller that behaves like the bang-bang control law, but that does not suffer from the adverse effects of chattering. Workman proposed to saturate the controller only when it is practical to do so, and, as the system approaches the reference point, the control law switches to a linear PD controller. As we proceed in this paper we shall revisit the PTOS, analyze its weaknesses and propose improvements where there is room to do so.

Another nonlinear approach designed to improve the performance of linear systems is provided by Lin et al., [1998] and was later generalized and expanded by Chen et al., [2003] under the name of Composite Nonlinear Feedback (CNF). This strategy proposes the design of a linear control law that provides the system with a small damping ratio for a fast rise time, associated with a nonlinear control that adds damping to the system as it approaches the reference point in order to eliminate the overshoot. This controller is capable of excellent results, however, once CNF does not explicitly include the input saturation in its design, the tuning process associated with the controller must necessarily be step-dependent in order to achieve a good performance. As a result, tuning this controller may become somewhat tedious and for very large steps the performance drops.

Other control methods that achieve a good performance for this class of systems include the LQG approach, Lewis and Syrmos, [1995], nonlinear PID control methods such as Su et al., [2005], sliding mode controllers, Utki., [1992], forms of Model Predictive Control (MPC), among many others. Some of these controller consider the saturation levels in the design process, and others do not. In the same form, some are switching controllers and others computationally demanding, but none of them achieve near or *quasi*-time optimal performance with a continuous control law. The proposed controller, on the other hand, presents no switching function and its tuning process is

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straightforward: only two parameters must be tuned and they are not step dependent, that is, one set of parameters performs extremely well for a wide range of step references. Furthermore, this is a saturation oriented controller: the control law is specifically designed to reach the saturation levels both at acceleration and deceleration.

The rest of the paper is organized as follows. Section 2 will present the model of interest along with a short but necessary discussion on the PTOS. Section 3 will present the proposed controller. Section 4 will expose simulation and experimental results and concluding remarks will be given in Section 5.

2. ON THE PROXIMATE TIME OPTIMAL SERVOMECHANISM

This section will be used to make some useful remarks on the structure of the PTOS, for it is on this controller that the proposed design stands. But first, let us present the model of interest.

2.1 Rigid Body Equations of Motion

The system in hand is comprised of a body of mass M subject to some friction f and disturbance d,

$$M\ddot{y} = u - f - d.$$

Due to the frequent but undesired presence of friction and disturbances, a model-based friction compensator must be employed. Once it is beyond the scope of this paper to provide a discourse on these compensators, the interest reader shall be referenced to the vast academic work in the are, such as the survey by Radke and Gao, [2006]. Such compensators are at a mature level and are commonly implemented in the literature: Zheng et al., [2009]; Salton et al., [2010a] and Salton et al., [2010b].

With the adverse effects of friction and disturbance overcome, the system is fully described by the rigid body equations of motion given by:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= b\,\mathrm{sat}(u) \\ y &= x_1 \end{aligned} \tag{1}$$

where x_1 and x_2 refer to the position and velocity, $b := 1/M$ and "sat" is the saturation function defined as:

$$\mathrm{sat}(z) = \begin{cases} \bar{u}, & z > \bar{u} \\ z, & |z| < \bar{u} \\ -\bar{u}, & z < -\bar{u} \end{cases} \tag{2}$$

with $\bar{u}$ the saturation level of the control input.

2.2 The Construction of a PTOS

As previously mentioned, time optimal performance for rigid body dynamics systems is achieved by the bang-bang controller, a switching controller that applies maximal acceleration followed by maximal deceleration. This control strategy may be described in a feedback structure by a switching curve given by,

$$\begin{aligned} u_{to}(t) &= \mathrm{sgn}(\sqrt{2b\bar{u}|e|} - x_2) \\ e &:= x_1 - y_r \end{aligned} \tag{3}$$

note that in this paper the sgn(·) function is defined as,

$$\mathrm{sgn}(z) = \begin{cases} \bar{u}, & z > 0 \\ 0, & z = 0 \\ -\bar{u}, & z < 0 \end{cases}. \tag{4}$$

Workman adapted the TOC law (3) to accommodate measurement noise and plant uncertainties. The PTOS design may be described in the following three different steps.

Step 1: The effects of chattering are minimized by eliminating the sgn(·) function (4) where possible, also, a free parameter k is applied in order to scale the control input,

$$u(t) = k(-f(e) - x_2),$$

where $f(e)$ is defined as

$$f(e) = \mathrm{sgn}(e)\sqrt{2b\bar{u}|e|}.$$

This control law is, in fact, a high gain that saturates the controller and drives the system to the time-optimal switching curve $x_2 = -f(e)$, i.e., full acceleration is achieved. However, when the system reaches the switching curve, the control input goes to zero. Another term must be added so that the input goes from one saturation level to another (from $\bar{u}$ to $-\bar{u}$ or vice-verse).

Step 2: Saturation of the controller during deceleration is achieved by adding the term $\mathrm{sgn}(e)\bar{u}$ to the nonlinear function,

$$u(t) = k(-f(e) - x_2) + \mathrm{sgn}(e)\bar{u}.$$

Or, in a more familiar form,

$$u(t) = \mathrm{sat}[k(-f_{pto}(e) - x_2)],$$

with $f_{pto}(e)$ defined as

$$f_{pto}(e) = \mathrm{sgn}(e)(\sqrt{2b\bar{u}|e|} - \bar{u}/k). \tag{5}$$

While this controller is able to saturate the system both during acceleration and deceleration, it is not able to asymptotically track the reference. In fact, the equilibrium point is given by:

$$\dot{y} = \dot{x}_2 = 0 \rightarrow u = x_2 = 0 \rightarrow f(e) = 0, \tag{6}$$

which implies

$$|e| = \frac{\bar{u}}{2bk^2}. \tag{7}$$

Step 3: Asymptotic stability is achieved by implementing a switching control law. As the system approaches the reference, the controller switches from the complex nonlinear function (5) to a simple Proportional Derivative (PD) controller. The cost of using such nonaggressive linear control law is that the PD controller is unable to prevent the system from overshooting. To overcome this problem, the so-called "acceleration discount factor" α was included in the original nonlinear function $f_{pto}(e)$, adding conservatism to the solution.

The control law becomes:

$$u(t) = k_2(-f_{ptos}(e) - x_2), \tag{8}$$

with,

$$f_{ptos}(e) = \begin{cases} (k_1/k_2)e, & \text{for } |e| \le y_l, \\ \mathrm{sgn}(e)(\sqrt{2b\alpha\bar{u}|e|} - \bar{u}/k_2), & \text{for } |e| > y_l. \end{cases} \tag{9}$$

A stability condition requires that $0 < \alpha < 1$, and the following constraints guarantee a continuous switching of the controller,

$$y_l = \frac{\bar{u}}{k_1}, \;\; k_2 = \sqrt{\frac{2k_1}{b\alpha}}. \tag{10}$$

□

While this discussion only scratches the surface of the PTOS it is sufficient for the understanding of the remainder of this paper. For full details the interested reader should refer to the relevant literature.

The main objective of this paper is to improve the performance of this controller by eliminating the switching function and the discount acceleration factor α. While theoretically α could take values arbitrarily close to one, in practice it must be somewhat conservative in order to prevent the system from overshooting. Notice that in the linear region, $|e| \le y_l$, the PD controller may be parameterized as,

$$K = \frac{1}{b}[4\pi^2\omega^2 \;\; 4\pi\omega\zeta], \tag{11}$$

with ω representing the undamped natural frequency of the system and ζ the damping ratio. As pointed out in Choi et al., [2006], the continuity conditions on the PTOS limit the damping ratio of the system. From (11) and (10) we have that,

$$\zeta = \sqrt{\frac{1}{2\alpha}},$$

and the larger the α, the smaller the damping and, consequently, the larger the overshoot. If α is pushed to its limit, $\alpha = 1$, then $\zeta = 0.707$ resulting in a large overshoot. In order to eliminate the necessity of the acceleration discount factor, two approaches might be taken: either an aggressive control law is proposed in order to replace the linear PD controller, Salton et al., [2011], or a different nonlinear control law is proposed in order to eliminate the switching function. The second approach is the one taken in this paper, where an elaborate nonlinear control law will be proposed in order to replace the switching function (8).

3. AN IMPROVED DESIGN

The following is useful result on the stability of system (1) under a general nonlinear controller. Without loss of generality, we assume $y_r = 0$ and the problem reduces to a stabilization problem of the equilibrium point $x := [x_1, x_2]^\intercal = 0$.

Lemma 1. Consider the closed-loop system composed of (1) and the control law

$$u = -h_1(x_1) - k_2 x_2, \tag{12}$$

where $h_1(\cdot)$ is a piecewise continuously differentiable function with $h_1(0) = 0$.

Suppose the following conditions [1] are satisfied for $x_1 \in \mathbb{T}$ with $\mathbb{T}$ a subset of $\mathbb{R}$:

A1: $h_1'(x_1) > 0$ and $\lim_{x_1 \to \pm\infty} h_1(x_1) = \pm\infty$;

A2: $k_2 > 0$;

A3: $\bar{u}\left(h_1' - k_2^2 b\right) < h_1' h_1 < \bar{u}\left(b k_2^2 - h_1'\right)$.

Then, the trajectory $x(t)$ of the closed-loop system satisfies $\lim_{t\to\infty} x(t) = 0$ if $x_1(t) \in \mathbb{T}, \forall t \ge 0$.

Proof: This proof is long and elaborate and shall be omitted. The full proof along with further details on the proposed design may be encountered in Salton et al., [2010b]. □

This result on itself provides a great deal of liberty when designing a controller for system (1) because $h_1(x_1)$ may be any nonlinear function that satisfies the conditions presented in the Lemma. With this in mind, we are now ready to present the main result of this paper: a nonswitching controller that outperforms the PTOS.

Theorem 2. Consider the closed-loop system composed of (1) and (12), with

$$\begin{aligned} h_1(x_1) &= k_1 \mathrm{sgn}(x_1)\left(\sqrt{2b\bar{u}\psi(x_1)|x_1|} - (\bar{u}/k_1)\psi(x_1)\right) \\ \psi(x_1) &= (1 - \mathrm{e}^{-\mu|x_1|}) \end{aligned} \tag{13}$$

for any

$$k_1 > 0, \;\; 2k_1^2 b/\bar{u} > \mu > 0. \tag{14}$$

Then, the closed-loop system is semi-globally asymptotically stable in the sense that, for any compact set $\mathbb{X}_o \subset \mathbb{R}^2$, there exists a $k_2 > 0$ (depending on $\mathbb{X}_o$), such that any trajectory with $x(0) \in \mathbb{X}_o$ satisfies $\lim_{t\to\infty} x(t) = 0$.

Proof: First, we note the function $h_1(\cdot)$ has the following properties. It is a continuously differentiable odd function with

$$h_1(x_1) = -h_1(-x_1), \;\; \lim_{x_1\to\pm\infty} h_1(x_1) = \pm\infty. \tag{15}$$

Its derivative satisfies $h_1'(x_1) = h_1'(-x_1)$,

$$\begin{aligned} h_1'(x_1) &= k_1\sqrt{b\bar{u}/2}\left(\frac{\psi(x_1) + x_1\psi'(x_1)}{\sqrt{x_1\psi(x_1)}}\right) - \bar{u}\psi'(x_1) \\ &= k_1\sqrt{\frac{b\bar{u}\psi(x_1)}{2x_1}} + k_1\sqrt{\frac{b\bar{u}x_1}{2\psi(x_1)}}\psi'(x_1) - \bar{u}\psi'(x_1) \\ &= k_1\sqrt{\frac{b\bar{u}\psi(x_1)}{2x_1}} - \frac{\bar{u}}{2}\psi'(x_1) \\ &\quad + \left(k_1\sqrt{\frac{b\bar{u}x_1}{2\psi(x_1)}} - \frac{\bar{u}}{2}\right)\psi'(x_1) > 0, \; \forall x_1 > 0 \end{aligned} \tag{16}$$

[1] We define $h_1'(x_1) := dh_1(x_1)/dx_1$ as the derivative of h_1. We drop the dependency of the functions on x_1 for ease of notation if it does not cause any confusion.

Furthermore,

$$h_1'(0) = \lim_{x_1\to 0} k_1\sqrt{\frac{b\bar{u}\psi(x_1)}{2x_1}} + k_1\sqrt{\frac{b\bar{u}x_1}{2\psi(x_1)}}\psi'(x_1) - \bar{u}\psi'(x_1)$$

$$= k_1\sqrt{\frac{b\bar{u}\mu}{2}} + k_1\sqrt{\frac{b\bar{u}}{2\mu}}\mu - \bar{u}\mu = k_1\sqrt{2b\bar{u}\mu} - \bar{u}\mu > 0.$$

In the last inequality of (16), we use the following facts:

(i) The inequality $|x_1|/\psi(x_1) \geq 1/\mu$ implies

$$k_1\sqrt{\frac{b\bar{u}x_1}{2\psi(x_1)}} \geq k_1\sqrt{\frac{b\bar{u}}{2\mu}} > \frac{\bar{u}}{2}.$$

(ii) The inequality

$$k_1\sqrt{\frac{b\bar{u}\psi(x_1)}{2x_1}} > \frac{\bar{u}}{2}\psi'(x_1)$$

is equivalent to

$$\frac{2k_1^2 b}{\bar{u}} > \frac{\psi'^2(x_1)x_1}{\psi(x_1)}$$

which holds if

$$\mu \geq \frac{\psi'^2(x_1)x_1}{\psi(x_1)}$$

or

$$\hbar(x_1) := 1 - e^{-\mu x_1} - \mu e^{-2\mu x_1}x_1 \geq 0.$$

It is true because $\hbar(0) = 0$ and

$$\hbar'(x_1) = \mu e^{-\mu x_1} - \mu e^{-2\mu x_1} + 2\mu^2 e^{-2\mu x_1}x_1 \geq 0.$$

For the remaining of the proof we need to define the unsaturated region $\mathbb{U}$ as,

$$\mathbb{U} = \{(x_1,\ x_2) \in \mathbb{R}^2 \mid |-h_1(x_1) - k_2x_2| \leq \bar{u}\},$$

and the subset $\mathbb{T} \subset \mathbb{R}$. We first show that, there exists a finite time T such that

$$x(T) \in \mathbb{U},\ |x_1(t)| \leq \bar{x}_1,\ |x_2(t)| \leq \bar{x}_2,\ \forall t \in [0,T] \quad (17)$$

for some constants $\bar{x}_1$ and $\bar{x}_2$ depending on $\mathbb{X}_o$. If $x(0) \in \mathbb{U}$, (17) is trivial with $T = 0$. Otherwise, we note that for $x(0) \in \mathbb{X}_o$ and a finite T, $\|x(t)\|$ is bounded for $t \in [0,T]$. Then, we can define a finite constant $x_1^* > 0$ as

$$\int_0^{x_1^*} h_1(y)dy = V([\bar{x}_1, \bar{x}_2]) = \int_0^{\bar{x}_1} h_1(y)dy + \frac{\bar{x}_2^2}{2b}. \quad (18)$$

and hence $\mathbb{T} = \{x_1 \in \mathbb{R} \mid |x_1| \leq x_1^*\}$. Clearly, we have $x_1^* \geq \bar{x}_1$.

It is ready to check the assumptions A_1-A_3 in Lemma 1. In fact, A_1 is satisfied from the aforementioned properties of $h_1(\cdot)$ and A_2 is self evident. It remains to examine A_3, that is

$$\bar{u}(h_1'(x_1) - bk_2^2) < h_1'(x_1)h_1(x_1) < \bar{u}(bk_2^2 - h_1'(x_1)).$$

Due to the symmetry, it suffices to shows

$$h_1'(x_1)h_1(x_1) < \bar{u}(bk_2^2 - h_1'(x_1)),\ \forall x_1^* \geq x_1 \geq 0.$$

It is true if k_2 is sufficiently large for

$$k_2^2 > h_1'(x_1)h_1(x_1)/(\bar{u}b) + h_1'(x_1)/b,\ \forall x_1^* \geq x_1 \geq 0.$$

It should be noted that $x_1^* \geq x_1$ is critical in the above inequality. Actually, its right hand side term approaches infinity as x_1 goes to infinity, so it is impossible to find a finite k_2 for the inequality for all $x_1 \geq 0$. Now, A_3 is satisfied.

What is left to show is that $x_1(t) \in \mathbb{T},\ \forall t \geq 0$. From the aforementioned definition of $\mathbb{T}$, $x_1(t) \in \mathbb{T}$ is true for $t \in [0,T]$ as shown in (17). For any $t > T$, the trajectory is inside $\mathbb{U}$, we have,

$$\int_0^{x_1(t)} h_1(y)dy \leq \int_0^{x_1(t)} h_1(y)dy + \frac{x_2(t)^2}{2b} = V(x(t)) < V(x(T)),$$

because,

$$\dot{V}(x) = h_1(x_1)x_2 + x_2(-h_1(x_1) - k_2x_2) = -k_2x_2^2 < 0.$$

On the other hand, (17) implies

$$|x_1(T)| \leq \bar{x}_1,\ |x_2(T)| \leq \bar{x}_2 \quad (19)$$

and hence

$$V(x(T)) \leq \int_0^{\bar{x}_1} h_1(y)dy + \frac{\bar{x}_2^2}{2b} = \int_0^{x_1^*} h_1(y)dy.$$

As a result, we have

$$\int_0^{x_1(t)} h_1(y)dy \leq \int_0^{x_1^*} h_1(y)dy$$

or $|x_1(t)| \leq x_1^*$, i.e., $x_1(t) \in \mathbb{T}$. The proof is thus complete. □

Remark: Notice that the main difference between the proposed controller and the traditional PTOS is on how the controllers guarantee the asymptotic stability of the system. In fact, the motivation for the proposed design comes from the discussion on PTOS given in Section 2, Step 2. Instead of proposing a switching function, as the original controller does, the proposed design includes the nonlinear function $\psi(\cdot)$ in order to achieve the equilibrium point at the origin. In other words,

$$\dot{y} = \dot{x}_2 = 0 \to u = x_2 = 0 \to h_1(x_1) = 0, \quad (20)$$

which implies

$$|x_1| = \frac{\bar{u}}{2bk^2}\psi(x_1). \quad (21)$$

Provided μ is chosen according to (14), this equality is only satisfied for $x_1 = 0$, and hence, the equilibrium point at the origin is achieved. ∎

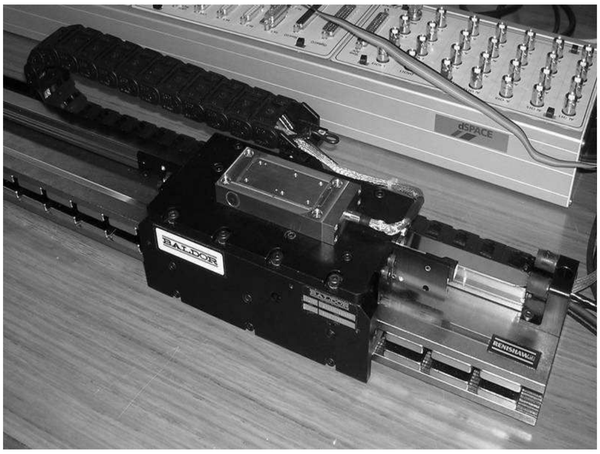

Fig. 1. Experimental set up of the electromagnetic motor.

4. SIMULATED AND EXPERIMENTAL RESULTS

In this Section simulated and Experimental comparisons between the proposed design, the TOC and the traditional PTOS will be made. Due to the impractical nature of TOC, this controller was only simulated, but a comparison between the implemented controllers and the simulated TOC is presented in order to demonstrate how close to the theoretical limit the proposed controller is.

The parameters of the system depicted in Fig. 1 and described as in (1) are $\bar{u} = 1$ and $b = 17000$, so that x_1 units are given in millimeters. The TOC controller is implemented via (3) and the PTOS parameters are given by,

$$k_1 = 2.09,\ \alpha = 0.7, \tag{22}$$

so that $k_2 = 0.019$ and the damping ratio is $\zeta = 0.85$.

The parameters chosen for the proposed controller, henceforth called *Quasi*-Time Optimal Servomechanism (QTOS), are given by,

$$k_1 = k_2 = 0.325,\ \mu = 36. \tag{23}$$

Notice the choice to fix $k_1 = k_2 = k$ is done to simplify the tuning process of the controller. By doing so, k becomes a free parameter used to scale the input and is extremely easy to be tuned online. In fact, in a practitioner point of view, the controller may be rewritten as,

$$\begin{aligned} u &= -k(h(x_1) + x_2) \\ h(x_1) &= \mathrm{sgn}(x_1)\left(\sqrt{2b\bar{u}\psi(x_1)|x_1|} - (\bar{u}/k)\psi(x_1)\right) \\ \psi(x_1) &= (1 - \mathrm{e}^{-\mu|x_1|}) \end{aligned} \tag{24}$$

Fig. 2 shows the normalized response y/y_r for steps of 1, 10, 25, 50 and 70 mm. One can clearly see that the performance achieved by the proposed controller is closer to the time-optimal one than the performance achieved by the traditional PTOS. This is even clearer in Figure 3, where we focused at the 70 mm response. The top plot shows the trajectories of the position and the bottom plot shows the three different inputs. Notice how the proposed QTOS input is very similar to that of the TOC, hence the aggressive performance achieved by the controller. In fact, the performance achieved by the proposed controller and that given by the time optimal one are indistinguishable in Fig. 3.

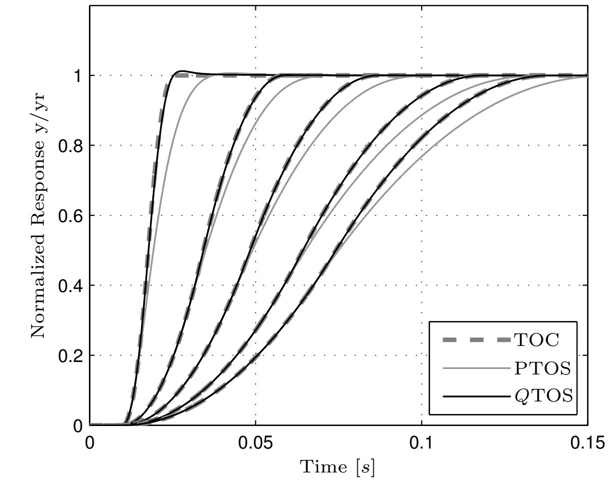

Fig. 2. Normalized simulated responses (y/y_r) for steps of 1, 10, 25, 50 and 70 mm for the three comparative controllers.

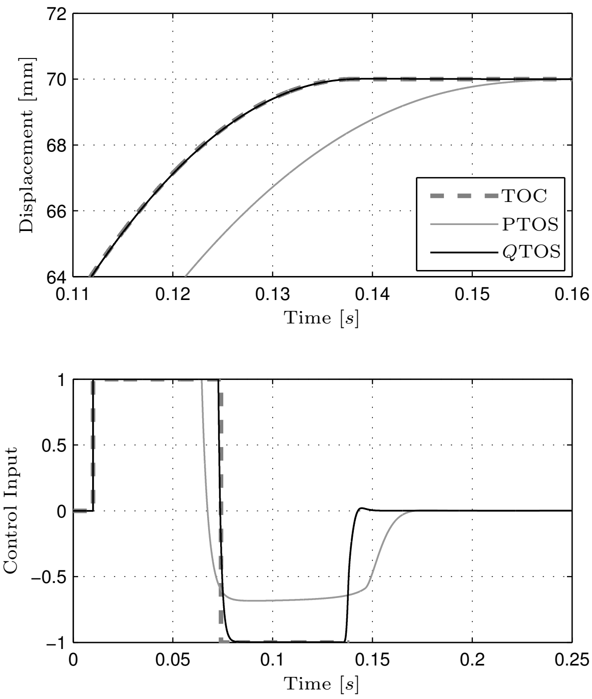

Fig. 3. Simulated response of TOC, PTOS and QTOS for a 70 mm step reference.

The next plots presented in Fig. 4 and Fig. 5 come from the actual plant response to the QTOS and the PTOS. As mentioned before, the TOC responses in these plots were obtained by simulation. In order to implement the controllers a DSP system (dSPACE-DS1103) with sampling frequency of 10 kHz was used. When tuning the control parameters a limited overshoot of 30 μm was imposed

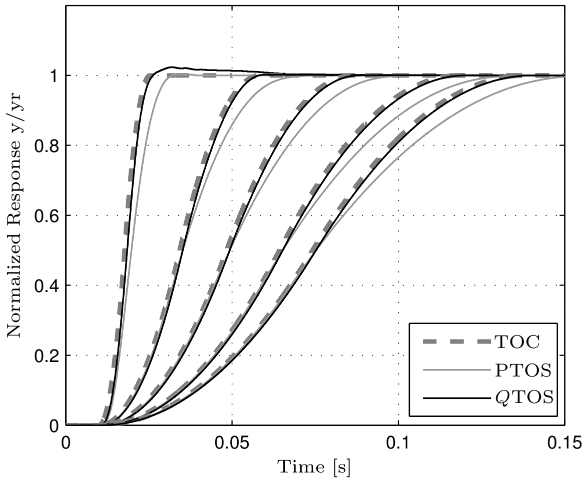

Fig. 4. Plant responses (y/y_r) for steps of 1, 10, 25, 50 and 70 mm for the three comparative controllers.

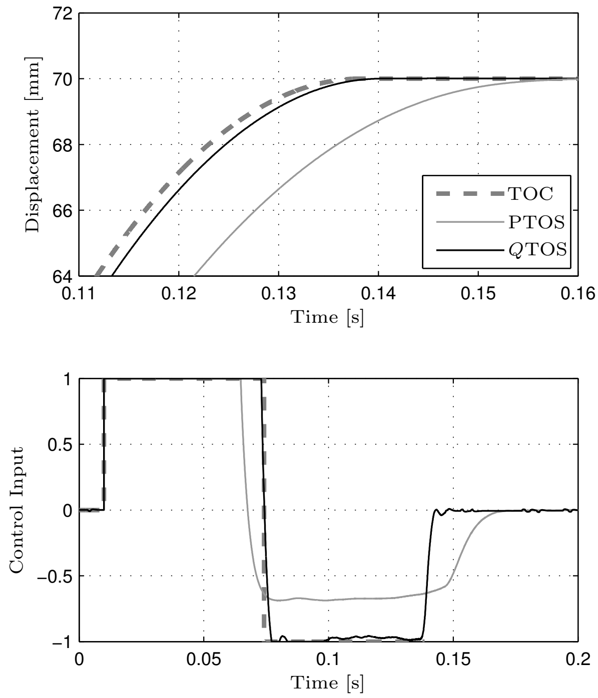

Fig. 5. Plant response of TOC, PTOS and QTOS for a 70 mm step reference.

independently of the step size (the controller parameters are the same as given in the simulation results). Moreover, a conventional state observer was necessary once only the position is available for feedback. It is important to emphasize that for all the step responses we have used the same set of parameters. This shows how simple it is to tune the proposed controller, which only has two parameters to be tuned, namely k and μ. Despite this tuning simplicity, the proposed controller is able to achieve truly *quasi*-time optimal performance.

5. CONCLUSION

A new form of near time optimal servomechanism was proposed in this paper. The proposed desgin adapts the original PTOS in order to eliminate the switching function present it that controller. Along with the switching function, the necessity for the acceleration discount factor and, hence, the conservatism present in the traditional control law, are eliminated. Simulation and experimental results have shown the effectiveness of the proposed design, which achieves performances comparable to that of the theoretical limits given by time optimal control.

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