2016 IEEE Conference on Control Applications (CCA)
Part of 2016 IEEE Multi-Conference on Systems and Control
September 19-22, 2016. Buenos Aires, Argentina

Quaternion-based Dynamic Control of a 6-DOF Stewart Platform For Periodic Disturbance Rejection

Paulo F. S. R. de Faria[1], Rafael S. Castro[1], Aurélio T. Salton[1] and Jeferson V. Flores[2]

***Abstract*— This paper proposes a simultaneous decoupled dynamic linear translational and non-linear rotational quaternion-based control of a Stewart platform. For the translation of the platform, a mixed design composed by $\mathcal{H}_\infty$, $\mathcal{D}$-stability and internal model control is presented. An augmented representation of the system allows the controller design to be cast as an optimization problem constrained by Linear Matrix Inequalities (LMI). For the rotational control of the end effector, a Lyapunov-LaSalle approach is used to guarantee asymptotic stability of the closed loop system. Numerical simulations are used to show that the final solution is able to stabilize the system around the reference vector and successfully reject external periodic perturbations.**

I. Introduction

The Stewart platform manipulator consists in a six degrees of freedom (6DOF) parallel kinematic system given by a closed-kinematic chain (CKC) mechanism. While this platform was originally designed as an aircraft simulator motion base [1], the CKC structure that it possesses have expanded its applicability to different areas [2]. In particular, when compared to open kinematic chain mechanisms, CKC manipulators have a higher structural rigidity, noncumulative actuator errors and a payload that is proportionally distributed to the links, granting a higher strength-to-weight ratio [3]. Therefore, there is significant interest in parallel manipulators in general and in the 6-DOF Stewart platform in particular [4], [5], [6], whose modern applications range from industrial-grade manipulators [7] to offshore cargo transfer mechanisms [8].

This paper considers the scenario where a Stewart platform is used as a stabilization device on the ocean as, for example, an offshore cargo transfer mechanism. In this case, it is desired that the effector (the top reference frame in Fig. 1) remains as steady as possible, negating the effects of waves perturbing the bottom frame. Obviously, the system is subject to external periodic and non-periodic perturbations, whose behavior and mathematical description are partly known. In order to minimize the effect of the unknown perturbations, the $\mathcal{H}_\infty$ norm from the perturbation to the output can be minimized. In the more specific case where it is known that the perturbations have some periodicity, a controller based on the Internal Model Principle (IMP) can be used [9]. In few words, the IMP states that if the closed-loop system is stable and if the nonvanishing modes of the disturbance signal are replicated by the control law, then asymptotic disturbance rejection is achieved. A resonant controller makes use of the IMP to successfully reject sinusoidal perturbations on a desired fundamental frequency [10]. In order to reject disturbances with higher harmonic content, multiple resonant controllers will be used in this paper.

In order to implement the multiple resonant control approach, this paper proposes a decoupled quaternion-based model of the Stewart platform. As a result, the task of controlling the position and the orientation of the platform may be performed separately and the dynamics describing the translational motion of the platform becomes that of a linear time invariant model. The linearity of this subsystem is explored and dynamic feedback controller is designed via an optimization problem subject to constraints in the form of Linear Matrix Inequalities (LMI). Finally, a non-linear controller able to minimize the effects of disturbances is applied to the rotational dynamics.

Notation: The ith component of vector x is defined as x_i, A^T denotes the transpose of matrix A and $\mathbf{I}$ denotes the appropriately sized identity matrix, I the 3×3 identity matrix and $\mathbf{0}$ the appropriately sized zero-filled matrix or vector. The operator $\otimes$ denotes the Kroenecker product.

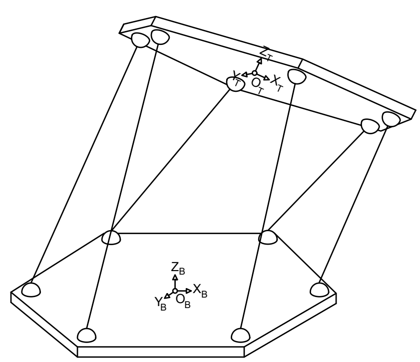

Fig. 1. The Stewart platform.

[1]Paulo de Faria, Rafael Castro and Aurélio Salton are with Faculty of Engineering, Pontifícia Universidade Católica do Rio Grande do Sul, Brazil `paulo.faria@acad.pucrs.br, rafael.castro@pucrs.br, aurelio.salton@pucrs.br`

[2]Jeferson Flores is with Faculty of Engineering, Universidade Federal do Rio Grande do Sul, Brazil `jeferson.flores@ufrgs.br`

978-1-5090-0755-4/16/$31.00 ©2016 IEEE

II. Quaternion-based Description of the Stewart Platform

The Stewart platform consists in a six degrees of freedom (6-DOF) parallel manipulator composed by a static base and a movable platform, which are linked by six variable-length actuators, as depicted Fig. 1.

A. Rigid Body Dynamics

The platform is a non-linear coupled system usually modeled using Lagrange or Newton-Euler formalism. Following the classic description of a generic 3D rigid body with respect to a coordinate frame whose origin coincides with the center of mass of the body, the Newton-Euler equations that represent the upper platform are given by

$$\begin{aligned} \tau(\omega, \dot{\omega}) &= I_m \dot{\omega} + S(\omega) I_m \omega, \\ F(\dot{v}) &= m\dot{v}. \end{aligned} \tag{1}$$

Here $\tau \in \mathbb{R}^3$ is the torque vector, $I_m \in \mathbb{R}^{3\times3}$ is the inertia tensor and $\omega \in \mathbb{R}^3$ is the angular velocity vector, all represented in the local body frame of the upper platform. Also, $F \in \mathbb{R}^3$ is the force vector and $v \in \mathbb{R}^3$ is the linear velocity vector, where these last two are represented in the global inertial frame, and m is the body mass of the end effector, whose center of mass is described by point O_T in Fig. 1. The term $S(\omega)I_m\omega$ represents the gyroscopic effect on the platform[1].

In order to relate the dynamics of the velocities, position and orientation of the upper platform, the following mapping will be used,

$$\begin{aligned} \dot{q}(\eta, \epsilon, \omega) &= \tfrac{1}{2}\begin{bmatrix} -\epsilon^T \\ \eta I + S(\epsilon) \end{bmatrix}\omega \\ \dot{p}(v) &= v, \end{aligned} \tag{2}$$

Here, $q(\eta, \epsilon) = [\eta \ \epsilon^T]^T \in \mathbb{R}^4$ is the body orientation unit quaternion [11] and $p = \begin{bmatrix} p_x & p_y & p_z \end{bmatrix}^T \in \mathbb{R}^3$ is the position vector of the end effector regarding the global inertial frame, with p_x, p_y and p_z related to the x-, y- and z-axis respectively.

Adding the gravity force on the system, the complete dynamics of the upper platform can then be expressed by

$$\dot{x} = \begin{bmatrix} \dot{q} \\ \dot{w} \\ \dot{p} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\begin{bmatrix} -\epsilon^T \\ \eta I + S(\epsilon) \end{bmatrix}\omega \\ {I_m}^{-1}(u_\tau + \tau_{ext} - S(\omega)I_m\omega) \\ v \\ m^{-1}(u_F + F_{ext}) + g \end{bmatrix}, \tag{3}$$

where $u_\tau \in \mathbb{R}^3$ and $\tau_{ext} \in \mathbb{R}^3$ are the input and external perturbation torques referenced to the local body frame, $u_F \in \mathbb{R}^3$ and $F_{ext} \in \mathbb{R}^3$ are the input and external perturbation forces referenced on the global inertial frame and g is the gravity vector. The external perturbations represent some important disturbances that the platform is subject to, such as: mass increase and center of mass shift in load conditions, external forces and torques applied directly on the base and top platforms, unmodelled elements and uncertain parameters.

B. Quaternion-based Jacobian

The Jacobian matrix J transforms the linear velocities of the actuators $\dot{l}$ to the linear and angular velocities of the platform, $\dot{p}$ and ω, respectively. This matrix can also be used in order to relate the linear forces of the six actuators $f = [f_1 \dots f_6]^T$ to the forces and torques applied on the top (F_T and τ_T) and bottom platform (F_B and τ_B), that is

$$F_x = \begin{bmatrix} F_T \\ \tau_T \\ F_B \\ \tau_B \end{bmatrix} = J^T f. \tag{4}$$

Consider the vectors involved in the inverse kinematics of the platform, shown on Fig. 2.

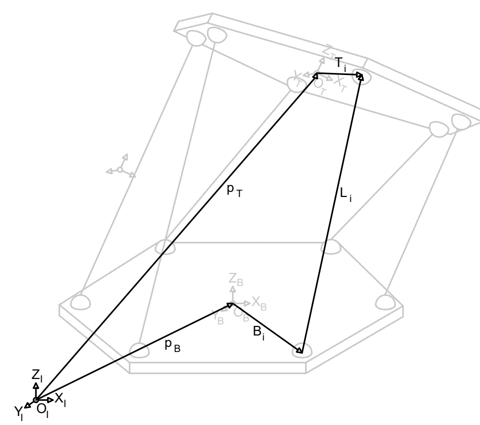

Fig. 2. Main vectors of the platform.

Let L_i be a vector of the same length and direction of the leg i, $i = 1 \dots 6$,

$$L_i = R_T^I T_i + p_T - (R_B^I B_i + p_B), \tag{5}$$

where R_T^I and R_B^I are the rotation matrices of the top and bottom platforms, respectively, both regarding the global inertial frame, and may be expressed as

$$R_j(\eta_j, \epsilon_j) = I + 2\eta_j S(\epsilon_j) + 2S^2(\epsilon_j), \quad j = T,\ B. \tag{6}$$

The vectors T_i and B_i are defined from the center of the top and bottom platforms, to the i_{th} top and bottom links, relative to the top and bottom platforms, respectively, and p_T and p_B are the position vectors of the top and bottom platforms, respectively.

Let also n_i be a unit vector with the same direction of the leg i, $i = 1 \dots 6$, so that $n_i = L_i/|L_i|$ and ω_T and ω_B

[1]In (1) and in further equations, the skew-symmetric matrix

$$S(x) = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}, \forall x \in \mathbb{R}^3.$$

is used to represent the vector cross product.

be the angular velocities of the top and bottom platforms in the local body frames. Differentiate both sides of (5) relative to time to obtain

$$\dot{L_i} = \dot{p_T} + S(\omega_T)(R^I_T T_i) - \dot{p_B} - S(\omega_B)(R^I_B B_i), \quad (7)$$

and define the velocity vector with the same direction of the i_{th} leg

$$\dot{l_i} = \dot{L_i} \cdot n_i. \quad (8)$$

Substitute (8) in (7) and apply a series of cross- and dot-product properties to obtain

$$\dot{l} = J \begin{bmatrix} \dot{p}_T \\ \omega_T \\ \dot{p}_B \\ \omega_B \end{bmatrix}, \quad (9)$$

where

$$J = \begin{bmatrix} n_1^T & (S(R^I_T T_1)n_1)^T & n_1^T & (S(R^I_B B_1)n_1)^T \\ \vdots & \vdots & \vdots & \vdots \\ n_6^T & (S(R^I_T T_6)n_6)^T & n_6^T & (S(R^I_B B_6)n_6)^T \end{bmatrix}. \quad (10)$$

From the energy conservation principle, if follows that the power P produced by F and τ must equal that produced by f, that is

$$P = F_x^T \begin{bmatrix} \dot{p}_T \\ \omega_T \\ \dot{p}_B \\ \omega_B \end{bmatrix} = f^T \dot{l}. \quad (11)$$

Substitute (9) in (11) and rearrange to obtain (4)[2].

III. Control Strategies

Two decoupled control strategies are proposed: a linear, dynamic translation controller and a non-linear rotation controller. To unify the outputs of both controllers, the Jacobian J in (10) is used to compute the inputs of the actuators. This approach simplifies the design of the controllers and allows for the use of a mixed solution for the linear and non-linear portions of the system.

A. Dynamic Translation Controller

The translation controller u_F acts on the Cartesian position p and linear velocities v of the platform, whose dynamics around the equilibrium point is represented by the system

$$S_1 := \begin{cases} \dot{x}_1 = Ax_1 + B_u u_F + B_\phi F_{ext} \\ z_1 = Cx_1 \end{cases} \quad (12)$$

where,

$$x_1 = \begin{bmatrix} p \\ v \end{bmatrix}, A = \begin{bmatrix} \mathbf{0} & I \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, B_u = B_\phi = \begin{bmatrix} \mathbf{0} \\ m^{-1} I \end{bmatrix}, \quad (13)$$

$$C = \begin{bmatrix} C_x^T & C_y^T & C_z^T & C_{\dot{x}}^T & C_{\dot{y}}^T & C_{\dot{z}}^T \end{bmatrix}^T = \mathbf{I},$$

[2]Provided the matrix $J^{-T} := (J^T)^{-1}$ is invertible around the equilibrium point, the controllers presented on Section III are able to generate the inputs of the linear actuators that effectively control the platform.

such that $x_1 \in \mathbb{R}^6$, $A \in \mathbb{R}^{6\times6}$, $B_u \in \mathbb{R}^{6\times3}$, $B_\phi \in \mathbb{R}^{6\times3}$ and $C \in \mathbb{R}^{6\times6}$.

The main goal of this control is to reject external sinusoidal perturbations. As such, the use of a resonant controller, which is based on the internal model principle (IMP), is the starting point of the proposed design. It is well known from the IMP that a perturbation signal can be asymptotically rejected if its dynamics are reproduced by the states of the controller.

If the periodic perturbation applied to the system is a sinusoidal signal of fundamental frequency σ_r, the control loop must include additional states in the form of

$$\begin{aligned} \dot{x}_r &= \bar{A}_r x_r + B_r e_r, \\ y_r &= x_r, \end{aligned} \quad (14)$$

where

$$\bar{A}_r = \begin{bmatrix} 0 & 1 \\ -(h\sigma_r)^2 & 0 \end{bmatrix}, B_r = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad (15)$$

e_r is the motion error, e.g., $r_x - p_x$, and $h = 1$ for the fundamental frequency and $h > 1 \in \mathbb{I}$ for the harmonics. In addition to rejecting the fundamental frequency of the perturbation, the second harmonic to this signal is also rejected. For this purpose, we define matrices A_r and A_h with $h = 1$ and $h = 2$ respectively. Since the platform has 3 axis of linear movement and the resonant controller has 2 states for each frequency at each axis, twelve states have to be introduced in the control loop. Furthermore, in order to deal with unknown load conditions, three extra "integrator states" are introduced in the controller.

To better define the proposed control loop, consider an augmented system S_a in the form of (12), where the matrices and vectors denoted with subscript a are the equivalent augmented counterparts of (13) given by

$$A_a = \begin{bmatrix} A & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -B_r C_x & A_r & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -B_r C_x & \mathbf{0} & A_h & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -B_r C_y & \mathbf{0} & \mathbf{0} & A_r & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -B_r C_y & \mathbf{0} & \mathbf{0} & \mathbf{0} & A_h & \mathbf{0} & \mathbf{0} \\ -B_r C_z & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & A_r & \mathbf{0} \\ -B_r C_z & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & A_h \\ -C_x & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -C_y & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -C_z & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix},$$

$$B_{u,a} = \begin{bmatrix} B_u & \mathbf{0} \end{bmatrix}^T, B_{\phi,a} = \begin{bmatrix} B_\phi & \mathbf{0} \end{bmatrix}^T, \quad (16)$$

such that $x_a \in \mathbb{R}^{21}$ encompasses the plant and controller states, $A_a \in \mathbb{R}^{21\times21}$, $B_{u,a} \in \mathbb{R}^{21\times3}$, $B_{\phi,a} \in \mathbb{R}^{21\times3}$. That is,

$$S_a := \begin{cases} \dot{x}_a = A_a x_a + B_{u,a} u_F + B_{\phi,a} F_{ext} \\ z_1 = C_a x_a \end{cases} \quad (17)$$

with $C_a = [C \ \mathbf{0}]$.

A state feedback is proposed in order to guarantee closed-loop stability of S_a. In this sense, consider the stabilization task as defined in Problem 1, together with additional performance criteria.

Problem 1. *Design a feedback gain K such that*

$$\dot{x}_a(t) = \mathbb{A}x_a \tag{18}$$

for $\mathbb{A} = (A_a + B_{u,a}K)$ is asymptotically stable and satisfy the following performance criteria:

PC1. *Place the closed loop eigenvalues λ_a of $\mathbb{A}$ inside a stable subregion $\mathcal{D}$ of the complex plane.*

PC2. *Minimize the $\mathcal{H}_\infty$ gain of the unknown perturbation F_{ext} to the output z_1, i.e., minimize*

$$\bar{\mu} = \sup_{\|F_{ext}\|_2 \neq 0} \frac{\|z_1(t)\|_2}{\|F_{ext}(t)\|_2} \tag{19}$$

The solution of Problem 1 subject the performance criteria **PC1** and **PC2** is presented in the next Theorem.

Theorem 1. *Consider the linear augmented system S_a and given constant matrices L and M defining an LMI region $\mathcal{D}$. If there are matrices $P = P^T = Q^{-1} > 0$ and Y with appropriate dimensions and a positive scalar $\mu > 0$ subject to the following constraints*

$$\begin{cases} L \otimes Q + M \otimes \Gamma(Q,Y) + M^T \otimes \Gamma(Q,Y)^T < 0, \\ \begin{bmatrix} \Gamma(Q,Y) + \Gamma(Q,Y)^T & \star & \star \\ B_{\phi,a}^T & -\mu^2 I & \star \\ C_a Q & \mathbf{0} & -\mathbf{I} \end{bmatrix} < 0 \end{cases} \tag{20}$$

with $\Gamma(Q,Y) = (A_a Q + B_{u,a} Y)$, then the control law $u_F = Kx_a$, with $K = YQ^{-1}$, solves Problem 1 and satisfies constraint **PC1**. *Furthermore, if the above inequalities are solved while minimizing μ,* **PC2** *is also satisfied.* □

The proof follows the same ideas presented in [12] and will be omitted due to space constraints.

B. Rotation Controller

The rotation controller acts on the angular position q and velocities w of the platform represented by the system

$$S_2 := \begin{cases} \begin{bmatrix} \dot{q} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \begin{bmatrix} -\epsilon^T \\ \eta I + S(\epsilon) \end{bmatrix} \omega \\ {I_m}^{-1}(u_\tau + \tau_{ext} - S(\omega) I_m \omega) \end{bmatrix}, \\ z_2 = \begin{bmatrix} q \\ w \end{bmatrix}, \end{cases} \tag{21}$$

by applying torques u_τ to the end effector. The control law objective is to maintain the system at the origin compensating the effects of external perturbations. We introduce such control law in the following theorem.

Theorem 2. *Consider the system S_2 and control law*

$$u_\tau = -k_1 \epsilon \operatorname{sgn}(\eta) - k_2 w \tag{22}$$

with positive scalars k_1 and k_2. The closed loop system is bounded-input bounded-ouput stable from τ_{ext} to ω with an arbitrarily small $\mathcal{L}_2$ gain. Furthermore, in the absence of external disturbances, control law (22) *achieves global asymptotic convergence of z_2 to the set $\mathcal{M} := \{z_2 \in \mathbb{R}^7 \mid z_2 = [\pm 1 \; \mathbf{0}]^T\}$.* □

Proof. Consider the Lyapunov function candidate

$$V(z_2) = k_1 \epsilon^T \epsilon + k_1 (1 - |\eta|)^2 + \tfrac{1}{2} \omega^T I_m \omega \tag{23}$$

and compute $\dot{V}(z_2)$ substituting the control law (22) to obtain,

$$\dot{V}(z_2) = -k_2 \omega^T \omega + \omega^T \tau_{ext}, \tag{24}$$

which is negative semi-definite for $\tau_{ext} = 0$. By defining the sign function such that $\operatorname{sgn}(x) = 1$ for nonnegative x and $\operatorname{sgn}(x) = -1$ otherwise, then

$$\omega \equiv 0 \Rightarrow \dot{\omega} \equiv 0 \Rightarrow \epsilon \equiv 0 \Rightarrow \eta \equiv \pm 1 \Rightarrow \dot{q} \equiv 0,$$

where we have used the property that $\eta^2 + \epsilon^T \epsilon = 1$. From LaSalle's invariance principle, it is clear that z_2 converges to $\mathcal{M}$, and from the fact that $V(z_2)$ is radially unbounded, it follows that this convergence holds globally.

To see that the closed loop system is $\mathcal{L}_2$ stable, take $V(z_2)$ as a storage function and note that (22) renders S_2 output strictly passive from τ_{ext} to w. This, in turn, implies bounded-input bounded-output stability with an $\mathcal{L}_2$ gain less than or equal to $1/k_2$ [13]. Finally, by making k_2 arbitrarily large one achieves an arbitrarily small $\mathcal{L}_2$ gain, which completes the proof.■

IV. Numerical results

The proposed control was simulated in a MATLAB environment on a platform modeled by (3) with parameters presented on Table I. The optimization problem presented in Theorem 1 was solved using YALMIP and SDPT3.

TABLE I

Platform and simulation parameters

Parameter	Symbol	Value
Sampling period $[s]$	T	0.01
Gravity $[m/s^2]$	g	9.85
Frequency of the perturbation [rad/s]	σ_p	0.4π
Top platform mass $[kg]$	m	1.36
Top platform tensor of inertia $[kgm^2]$	I_m	$\begin{bmatrix}1.705\times10^{-4}\\1.705\times10^{-4}\\3.408\times10^{-4}\end{bmatrix}$
Top platform radius $[mm]$	r_T	125
Gap between two actuators $[rad]$	a_T	$\pi/2$
Initial position, top platform $[mm]$	p_o	$\begin{bmatrix}0 & 0 & 180\end{bmatrix}^T$
Initial orientation, top platform	q_o	$\begin{bmatrix}1 & 0 & 0 & 0\end{bmatrix}^T$

A. Simulation Procedure

The developed environment applies a linear and angular perturbation at the center of mass of the bottom reference frame:

$$p_d = \begin{bmatrix}0\\0\\5\sin(w_p)\end{bmatrix}, r_d = \begin{bmatrix}0.125\sin(0.5w_p)\\0.25\sin(w_p)\\0.1\sin(0.2w_p)\end{bmatrix} \qquad (25)$$

where r_d is the Euler angle equivalent to q_d that is applied on the system. These disturbances are then naturally propagated to the top platform. For comparison purposes, Fig. 3 shows the system without control, considering that the actuators apply just the necessary reaction to gravity, where e_1, e_2 and e_3 are the position errors relative to $p_T = \begin{bmatrix}p_x & p_y & p_z\end{bmatrix}^T$ and α, β and γ are the equivalent Euler angles related to the rotation of the top platform.

B. Controlled System

In order to evaluate the system response to unknown load conditions, an added mass of 6.5 kg was applied to the end effector. The resulting performance is depicted in Fig. 4 where it is clear that, due to the IMP, the LMI-based control on the translational motion is capable of asymptotically rejecting the periodic and constant perturbations applied to the system. The nonlinear controller applied to the orientation of the platform shows a good performance, reducing the rotational perturbations by more than hundred fold. It is important to emphasize that if the system had no disturbances applied to it, the orientation error would also convergence asymptotically.

The resulting linear actuator forces depicted on Fig. 5 are obtained through the Jacobian J^{-T}. Once in steady-state, the actuators present a low magnitude response, this is so because the proposed controllers are not high-gain controllers. In particular, the position controller is based on the IMP and, therefore, can achieve robust asymptotic rejection of the disturbances without relying on excessive input efforts.

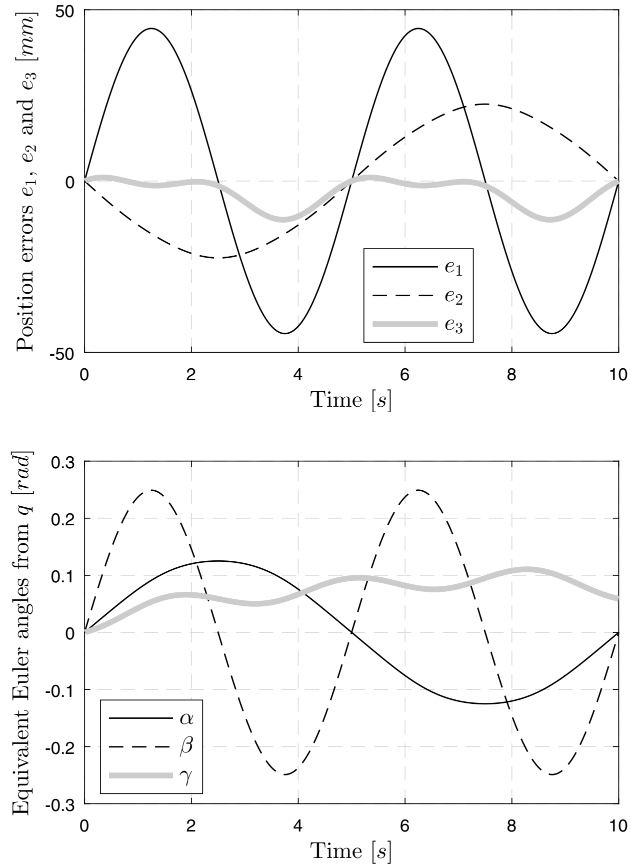

Fig. 3. States of the uncontrolled top platform subject to periodic perturbations caused by the movements of the base.

V. Conclusion

This paper proposed a decoupled, quaternion based dynamic model of the Stewart platform and a control method that leverages the mathematical independence of both models, allowing for two very different approaches to be used on each subsystem. The quaternion based, global inertial frame referenced Jacobian of the system was also presented, describing a simple form of coupling the outputs of both controllers. This technique avoids any linearization, thus achieving an improved performance.

The proposed dynamic controller uses the IMP to successfully reject periodic disturbances and constant disturbances acting on the translational motion of the platform. Furthermore, additional performance criteria are met by achieving $\mathcal{D}$-stability and $\mathcal{H}_\infty$ norm minimization. Since the design method uses LMIs, this approach has the benefits of providing a systematic way for designing the controller, as well as allowing for model uncertainties to be compensated (a theme to be addressed in future works). The Lyapunov-LaSalle based orientation control guarantees asymptotic stability of the unperturbed system and orbital stability for the perturbed system.

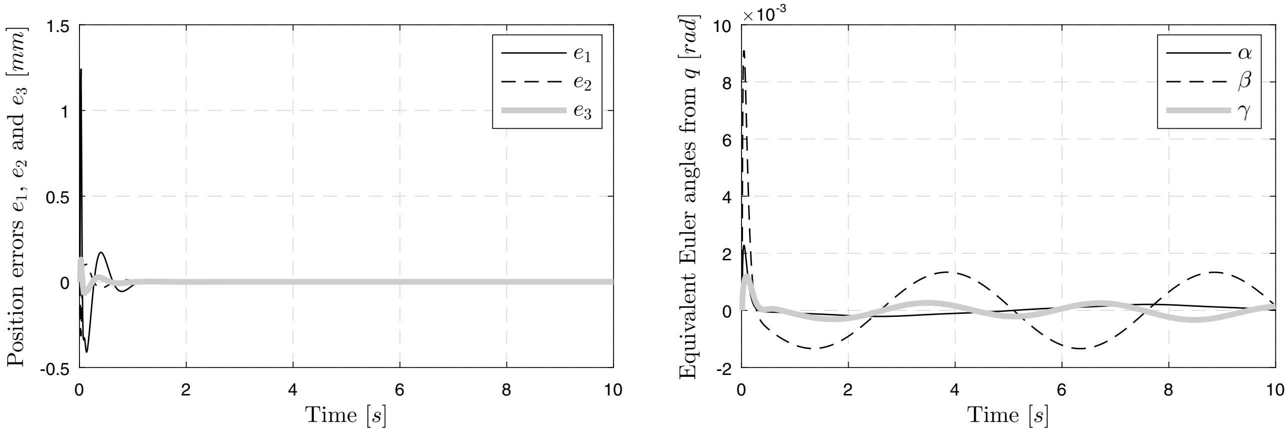

Fig. 4. States of the controlled top platform.

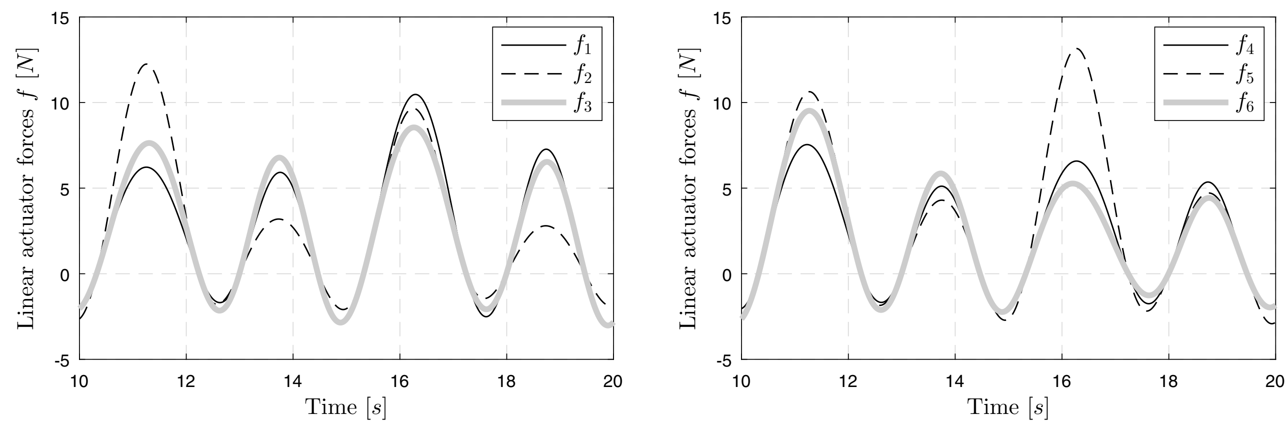

Fig. 5. Linear actuator forces effectively controlling the orientation and position of the top platform.

References

[1] D. Stewart, "A platform with six degrees of freedom," *Proc. Inst. Mech. Engr.*, vol. 180, no. 1, pp. 371–386, 1965.

[2] E. Fichter, "A stewart platform-based manipulator: general theory and practical construction," *The International Journal of Robotics Research*, vol. 5, no. 2, pp. 157–182, 1986.

[3] K. Liu, G. Lebret, and F. L. Lewis, "Dynamic analysis and control of a stewart platform manipulator," *Journal of Robotic Systems*, vol. 10, no. 5, pp. 629–655, 1993.

[4] M. Ayas, E. Sahin, and I. Altas, "Trajectory tracking control of a stewart platform," *Power Electronics and Motion Control Conference and Exposition (PEMC)*, vol. 16, pp. 720–724, 2014.

[5] P. Kumar, A. Chalanga, and B. Bandyopadhyay, "Position control of stewart platform using continuous higher order sliding mode control," *Asian Control Conference (ASCC)*, vol. 10, pp. 1–6, May 2015.

[6] S. Maged, A. Abouelsoud, and A. Fath El Bab, "A comparative study of unscented and extended kalman filter for position and velocity estimation of stewart platform manipulator," *Cyber Technology in Automation, Control, and Intelligent Systems (CYBER), 2015 IEEE International Conference on*, pp. 1216–1222, June 2015.

[7] K. H. Hunt, *Kinematic geometry of mechanisms.* Oxford University Press, 1978.

[8] H. Gonzalez, M. S. Dutra, and O. Lengerke, "Direct and inverse kinematics of stewart platform applied to offshore cargo transfer simulation," *13th World Congress in Mechanism and Machine Science*, 2011.

[9] L. Sainz, J. J. Mesas, and A. Ferrer, "Characterization of nonlinear load behavior," *Electric Power Systems Research*, vol. 78, no. 10, pp. 1773–1783, 2008.

[10] A. T. Salton, J. V. Flores, L. F. A. Pereira, and D. F. Coutinho, "A resonant-repetitive control scheme applied to uninterruptible power supplies (ups)," *Journal of Control, Automation and Electrical Systems*, vol. 24, no. 3, pp. 253–262, 2013.

[11] J. Yuan, "Closed-loop manipulator control using quaternion feedback," *IEEE Journal on Robotics and Automation*, vol. 4, no. 4, pp. 434–440, 1988.

[12] M. Chilali, P. Gahinet, and P. Apkarian, "Robust pole placement in lmi regions," *Automatic Control, IEEE Transactions on*, vol. 44, no. 12, pp. 2257–2270, Dec 1999.

[13] H. Khalil, "Nonlinear systems (3rd) prentice hall," *Upper Saddle River, NJ*, 2002.