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A Comparative Analysis of Repetitive and Resonant Controllers to a Servo-Vision Ball and Plate System ⋆

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Abstract: This paper presents a comparative analysis of two alternative control strategies based on the Internal Model Principle: the Repetitive Controller and the Multiple Resonant Controller. The proposed implementations are evaluated experimentally in a servo-vision ball and plate balancing system. The plant is composed by an orientable platform with a free rolling sphere on top, where the controlled variable is the ball position. The methodology considered to synthesize the associated state gains parameters is the Linear Quadratic Regulator approach. The experimental results compare characteristics such as steady-state error, transient response and input signal for each implemented control strategy.

Keywords: Ball and Plate System; Internal Model Principle; Repetitive Controller; Multiple Resonant Controller; Linear Quadratic Regulator.

1. INTRODUCTION

Due to increasing demands on productivity and quality, the precise control of dynamical systems has become a challenging and critical task. In this context, one of the most important problems is tracking periodic references and rejecting periodic disturbances. This scenario is verified, for example, in robotic manipulators [Liuzzo and Tomei, 2008] and any machinery performing repetitive operations [Pipeleers et al., 2008].

The Internal Model Principle (IMP) [Francis and Wonham, 1976] offers a solution to track periodic references and reject periodic disturbances. The IMP says that the controller (or the plant itself) should contain the critically stable and unstable modes of the signals to be tracked/rejected. In this context, the Resonant Controller [Chen, 1999] presents an internal model structure for compensation of sinusoidal oscillating modes. The Repetitive Controller approach [Yamamoto and Hara, 1988] is an alternative way to tackle this problem, based on the placement of infinite modes on imaginary axis of s complex plane with a single time-delay element in a positive feedback loop (where the delay is equals to the fundamental period of the signal). In the present work, the problem of tracking periodic references is addressed to discrete-time systems. In contrast to the continuous time, the discrete version of the Repetitive Controller places a finite number of poles equally spaced on unit circle of z complex plane (where the number of poles is equal to the rate between the period of the signal and the sampling time) [Flores et al., 2013].

In order to synthesize IMP-based control architectures, a common approach in the bibliography is to consider Linear Matrix Inequality (LMI) developments for robust placement of closed-loop poles [Salton et al., 2013]. An alternative synthesis methodology for Repetitive and Resonant Controllers is the Linear Quadratic Regulator (LQR) theory, based on energy minimization of system variables [Montagner et al., 2003].

The ball and plate balancing system is a commonly employed plant for control theory practical experimentation [Ker et al., 2007]. It consists in a platform with two rotational degrees of freedom and a free rolling sphere on top. The objective is regulate the ball position by commanding the actuators responsible for tilting the plate. A vast diversity of control theories have been previously implemented on a ball and plate system such as Back-stepping Control [Moarref et al., 2008], Sliding-Mode Control [Park and Lee, 2003] and Auto-Disturbance Rejection Controller (ADRC) [Duan et al., 2009]. Despite the diversity on the studies, the task of tracking the sphere according to periodic references with minimum error remains a topic little explored in the literature.

In this work we compare IMP-based theories such as Repetitive Controller and Multiple Resonant Controller in a servo-vision ball and plate plant. The implemented controllers aim to regulate the sphere position to track periodic references. The main objective is to analyze characteristics on the response of each implemented control strategy.

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2. PLANT DESCRIPTION

This section will characterize the experimental ball and plate servo-vision plant used in this research. The system main devices and the modeling of dynamics and kinematics will be presented.

2.1 System Devices

The ball and plate plant used for the experiment (Figure 1) is equipped with two servo-motors (Hextronik HXT12K) responsible for orienting an acrylic plate in pitch and roll angles. On top of this plate there is a free rolling sphere. The prototype is also equipped with an webcam (Microsoft VX-800) that acquire real-time images of the system for sensing the ball X and Y position on the plate.

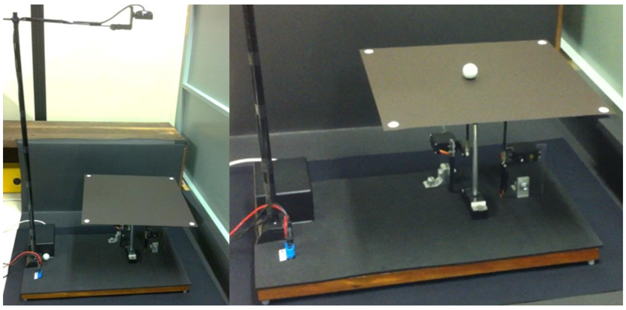

Fig. 1. Ball and plate system used on the experiment.

All image processing routines and control algorithms are executed directly in the software MATLAB on a PC. In order to command the servo-motors, a MATLAB script compute the desired angles and send them via serial communication to an Arduino prototyping board. This device generates the required PWM signals for the actuators.

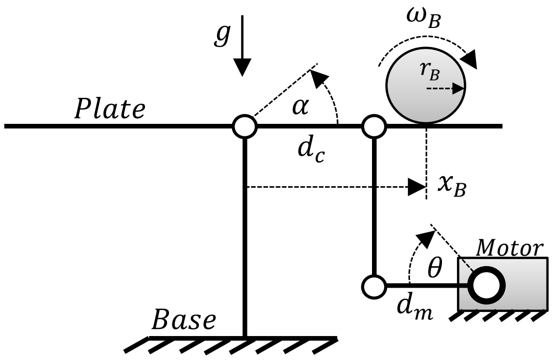

Fig. 2. Schematic of the ball and plate X axis subsystem.

2.2 Dynamical Modeling

To properly design the proposed controllers a state-space model of the system was obtained. The modeling procedure represents the two-dimensional ball and plate system as two uni-dimensional decoupled ball and beam systems: one about the X axis and other about the Y axis. The schematic on Figure 2 represents the X axis subsystem and its correspondent servo-motor. The subsequent mathematical formulation will be conducted only about the X subsystem variables, since the Y axis subsystem presents an identical structure.

The sphere equation of motion can be obtained using Lagrangian formalism by derivation of the following generic expression

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = Q_i \, . \tag{1}$$

This method consists in finding a Lagrangian $L(\dot{q}_i, q_i) = T(\dot{q}_i, q_i) - V(q_i)$ as a function of the system generalized coordinates q_i and their derivatives $\dot{q}_i$, where T represents the system total kinetic energy and V the total potential energy. The term Q_i in (1) represents the composite force actuating on q_i.

The total kinetic energy T of the sphere can be determined by the sum of translational kinetic energy T_t and rotational kinetic energy T_r. The translational energy is a function of the sphere mass m_B and its linear velocity $\dot{x}_B$ by $T_t = \frac{1}{2}\, m_B\, \dot{x}_B{}^2$. On the other hand, the rotational energy is a function of the sphere moment of inertia I_B and its angular velocity ω_B, according to $T_r = \frac{1}{2}\, I_B\, \omega_B{}^2$. Given the sphere radius r_B, it is possible to rewrite T_r as a function of the linear velocity, using the equality $\omega_B = \dot{x}_B / r_B$.

The total potential energy V is determined by the gravitational force incidence on the system, according to the equation $V = m_B\, g\, x_B\, \sin(\alpha)$, where x_B is the sphere position on the plate, g is the constant of gravity acceleration and α the plate tilt angle.

Based on the previous definitions, the relation that describes the sphere motion can be derived from (1) for $q_i = x_B$ and $Q_i = F_x$:

$$\frac{7}{5}\, \ddot{x}_B + g\, \sin(\alpha) = \frac{F_x}{m_B} \, .$$

Since the plate working tilt range is small ($\pm 15°$), it is valid to assume that $\sin(\alpha) \approx \alpha$ for obtaining a linearized equation.

The force F_x actuating on the system can be modeled as a linear friction by the relation $F_x = -f_c\, \dot{x}_B$, where f_c is the coefficient of kinetic friction between the plate and the sphere.

The schematic on Figure 2 also presents the articulated arm that transfers the servo-motor θ rotation to the plate rotation α in the X axis subsystem. The prototype presents an identical mechanism about the Y axis subsystem with another servo-motor. By simple trigonometry it is possible to write the relation $d_m \sin(\theta) = d_c \sin(\alpha)$ where d_m is the servo-motor arm length and d_c the distance between the plate central joint and the vertical arm joint. Approximating again the sine functions as its arguments yields a linearized kinematic relation $\theta = \kappa\, \alpha$, where the constant $\kappa = d_c / d_m$.

A generic second order relation $\ddot{\theta} + 2\, \zeta\, \omega_n\, \dot{\theta} + \omega_n{}^2\, \theta = \omega_n{}^2\, \theta_r$ is assumed for the servo-motors dynamics, where θ_r is the reference servo angle, ω_n is the servo-motor natural frequency and ζ is the servo-motor damping ratio. Using the linearized kinematic relation discussed above, we can also write $\ddot{\alpha} + 2\, \zeta\, \omega_n\, \dot{\alpha} + \omega_n{}^2\, \alpha = \omega_n{}^2\, \alpha_r$, where $\alpha_r = \theta_r / \kappa$ is the reference plate angle.

The system differential equations can be arranged on the continuous state-space standard

$$\begin{aligned} \dot{x}(t) &= \mathcal{A}\,x(t) + \mathcal{B}\,u(t)\,, \\ y(t) &= \mathcal{C}\,x(t)\,, \end{aligned}$$

where $x = [x_B \;\; \dot{x}_B \;\; \alpha \;\; \dot{\alpha}]'$ are the system states, the input $u = \alpha_r$ is the reference plate angle and $y = x_B$ denotes the system output. The matrices $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ are respectively:

$$\mathcal{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{5}{7}\frac{f_c}{m_B} & -\frac{5}{7}g & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -{\omega_n}^2 & -2\zeta\omega_n \end{bmatrix}, \quad \mathcal{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ {\omega_n}^2 \end{bmatrix}, \quad \mathcal{C} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}'. \tag{2}$$

The system dynamics can also be represented in the discrete state-space framework:

$$\begin{aligned} x(k+1) &= A\,x(k) + B\,u(k)\,, \\ y(k) &= C\,x(k)\,. \end{aligned} \tag{3}$$

The equivalent matrices A and B for discrete representation can be evaluated using the exact discretization method [DeCarlo, 1995], by numerically computing $\Phi = e^{T_s \Omega}$ where T_s is the plant sampling period and Φ e Ω are:

$$\Phi = \begin{bmatrix} A & B \\ 0 & I \end{bmatrix}, \qquad \Omega = \begin{bmatrix} \mathcal{A} & \mathcal{B} \\ 0 & 0 \end{bmatrix}.$$

Using this methodology, the output equation term C in discrete representation will be equal to the $\mathcal{C}$ in continuous format.

3. CONTROL DESIGN

This section presents a generic design procedure of a discrete control system for tracking periodic references and rejecting periodic disturbances, both with harmonic content. Two different strategies will be proposed in this context: the Repetitive Control and the Multiple Resonant Control. It will be also presented a Linear Quadratic Regulator synthesis procedure compatible with both control strategies mentioned.

3.1 Internal Model Principle

Let a generic discrete time Internal Model Controller (IMC) be defined as

$$\begin{aligned} \xi(k+1) &= A_c\,\xi(k) + B_c\,e(k) \\ e(k) &= r(k) - y(k) \end{aligned} \tag{4}$$

where $\xi(k)$ is the controller state vector and $e(k)$ the output system error. The $r(k)$ signal is defined as reference for the output $y(k)$.

According to the Internal Model Principle (IMP) [Francis and Wonham, 1976], tracking a periodic signal $r(k)$ and/or rejecting a periodic disturbance $d(k)$ in a linear plant is guaranteed if the controller or the plant contains the marginally stable and unstable modes of the signals to be tracked and/or rejected. This statement assumes the closed-loop system is stable.

The basic concept of the Repetitive Controller is that any periodic signal can be generated by a system including a time-delay element $z^{-\tau}$ in a positive feedback loop. The samples of delay τ must correspond, in this case, to the fundamental discrete period of the reference/disturbance signal. The Repetitive discrete transfer function $G_r(z)$ is implemented as follows:

$$G_r(z) = \frac{1}{1 - z^{-\tau}}\,.$$

Note that $G_r(z)$ contains τ equally spaced modes on the unit circle of z complex plane [Flores et al., 2013].

The Repetitive Controller $G_r(z)$ transfer function can also be implemented to the state-space framework (4) with the following definitions [Freeman et al., 2008]:

$$A_c = \begin{bmatrix} 0_{(1\times\tau-1)} & 1 \\ I_{(\tau-1)} & 0_{(\tau-1\times1)} \end{bmatrix}, \qquad B_c = \begin{bmatrix} 1 \\ 0_{(\tau-1\times1)} \end{bmatrix},$$

where $A_c \in \mathbb{R}^{\tau\times\tau}$ and $B_c \in \mathbb{R}^{\tau\times1}$.

The main idea behind the Multiple Resonant Controller is to generate sinusoidal oscillating dynamics for composing the fundamental frequency and harmonics of the signal of interest. In order to implement a discrete Multiple Resonant Controller, it is convenient to start from its continuous state-space format

$$\dot{\xi}(t) = \mathcal{A}_c\,\xi(t) + \mathcal{B}_c\,e(t)$$

that is given by [1]:

$$\begin{aligned} \mathcal{A}_c &= \mathrm{diag}\{F(\omega_1), F(\omega_2), \cdots, F(\omega_N)\} \\ \mathcal{B}_c &= \begin{bmatrix} G' \; G' \; \cdots G' \end{bmatrix}' \end{aligned}$$

Note that the terms $\mathcal{A}_c \in \mathbb{R}^{2N\times2N}$ and $\mathcal{B}_c \in \mathbb{R}^{2N\times1}$ represents the combination of resonance dynamics for the reference/disturbance harmonics $\omega_h = h\,\omega_0$, $h = 1, 2, \ldots, N$ where the fundamental frequency ω_0 is determined by $\omega_0 = 2\pi/(T_s\tau)$.

The individual resonance dynamics for each frequency ω_h is given by the terms $F(\omega_h)$ and G with the following structure [Pereira et al., 2013]:

$$F(\omega_h) = \begin{bmatrix} 0 & 1 \\ -{\omega_h}^2 & 0 \end{bmatrix}, \qquad G = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

The equivalent matrices A_c and B_c for discrete representation according to (4) can be evaluated using a discretization technique. One option is to apply the discretization procedure presented on Section 2 to obtain a discrete equivalent of the plant.

3.2 Linear Quadratic Regulator Synthesis

For matching the stability condition and transient constraints, the system closed-loop modes can be arranged by the linear control law

$$u(k) = -K\,x(k) - K_c\,\xi(k)\,, \tag{5}$$

[1] diag{ A , B } denotes a diagonal matrix obtained from elements A and B.

where K is a vector with the feedback gains of the plant states $x(k)$ and K_c is a vector with the feedback gains of the controller states $\xi(k)$. In order to apply the Linear Quadratic Regulator methodology for synthesizing these feedback gains it is necessary to find and augmented state-space representation of the system. This procedure can be done by defining and augmented state vector $z(k)$ that contains the plant states and the controller states as $z(k) = \left[x(k)' \ \xi(k)'\right]'$.

The augmented dynamics of $z(k)$ in the state-space format

$$z(k+1) = A_A \ z(k) + B_A \ u(k) + E_A \ r(k)$$

is determined with algebraic manipulation, leading to the following definitions:

$$A_A = \begin{bmatrix} A & 0 \\ -B_c C & A_c \end{bmatrix}, \quad B_B = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad E_A = \begin{bmatrix} 0 \\ B_c \end{bmatrix}.$$

The terms A_c and B_c that comes from the controller model can be either a Repetitive or a Multiple Resonant Controller. Note also that A, B and C comes from the plant model, according to (3).

The control law previously defined in (5) can also be represented in the subsequent augmented form

$$u(k) = -K_A \ z(k) \ ,$$

where the augmented feedback gains vector K_A is constructed as

$$K_A = [K \ K_c] \ .$$

Now, let we define a Linear Quadratic Regulator Functional $\mathcal{J}$ as

$$\mathcal{J} = \sum_{k=0}^{\infty} \Big(z(k)'Q_A \, z(k) + u(k)'R \, u(k) \Big) \ .$$

where Q_A penalizes the energy on the augmented states $z(k)$ and R penalizes the energy on the system inputs $u(k)$.

The augmented penalty matrix $Q_A = \text{diag}\{ \ Q \ , \ Q_c \ \}$ combines the plant penalty matrix Q and the controller penalty matrix Q_c such that $Q = \text{diag}\{ \ q_1 \ , \ q_2 \ , \ \dots \ , \ q_n \ \}$ where q_i is the penalty on the energy of the i^{th} plant state and n is the number of plant states. The controller penalty Q_c matrix is specified as $Q_c = \text{diag}\{ \ q_{c1} \ , \ q_{c2} \ , \ \dots \ , \ q_{cm} \ \}$, where q_{ci} is the penalty on the energy of the i^{th} controller state and m the number of controller states. Finally, the control penalty is defined as $R = \text{diag}\{ \ r_1 \ , \ r_2 \ , \ \dots \ , \ r_p \ \}$ where r_i is the penalty on the energy of the i^{th} control signal and p is the number of control inputs.

The optimal augmented feedback gains K_A that minimizes $\mathcal{J}$ [Dorato et al., 1995] is obtained by

$$K_A = \left(R + B_A{}'PB_A\right)^{-1} B_A{}'PA_A \, ,$$

where P is the solution of the following Discrete Algebraic Riccati Equation (DARE):

$$P = A_A{}' \left(P - PB_A \left(R + B_A{}'PB_A\right)^{-1} B_A{}'P \right) A_A + Q_A \, .$$

Note that a numerical algorithm must be adopted for finding P solution. In this work, the function `dare` on the software MATLAB was used for computing it.

4. RESULTS

This section presents the experimental implementation of the control methodology developed on Section 3 for the ball and plate servo-vision system described on Section 2.

First all the numerical setup parameters and the proposed scenario for testing the controllers will be shown. Then, the experimental results will compare characteristics, such as transient response, steady-state error and control input, for each implemented control strategy.

4.1 Experimental Setup

The focus of this work is making the sphere follow periodic trajectories. The reference $r(k)$ selected for this purpose has triangular waveform with the characteristics exposed on Table 1.

Table 1. Characteristics of the reference signal $r(k)$.

Waveform	Triangular
Amplitude	25% of plate width
Offset	Plate center
Discrete Period (τ)	100 samples
Fundamental Frequency (ω_0)	1.8589 rad/s

The plant continuous state-space model terms $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ were constructed using the parameters on Table 2 on the definitions in (2). Then, it was applied the exact discretization method for evaluating the correspondent discrete model terms A, B and C presented below. The sampling period considered is $T_s = 33.8$ ms, were 33.4 ms comes from the webcam acquisition period and 0.4 ms from the upper bound processing time.

$$A=\begin{bmatrix} 1 & 0.033 & -1.156 & -0.008 \\ 0 & 0.977 & -66.11 & -0.588 \\ 0 & 0 & 0.838 & 0.011 \\ 0 & 0 & -6.590 & 0.015 \end{bmatrix}, \ B=\begin{bmatrix} -0.043 \\ -4.547 \\ 0.162 \\ 6.590 \end{bmatrix}, \ C=\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}'.$$

Table 2. Model parameters.

Symbol	Value	Symbol	Value
m_B	0,02 kg	f_c	0,02 Ns/m
r_B	0,0125 m	g	9,81 m/s^2
ω_n	15 rad/s	d_m	0,03 m
ζ	1	d_c	0,05 m

Remember that the plant modeling procedure represents the ball and plate system as two uni-dimensional identical subsystems. So, all the implemented controllers had to be replicated for controlling the sphere X and Y positions on the plate.

The Repetitive Controller matrices A_c and B_c were constructed with the fundamental discrete period of the reference τ. For the Multiple Resonant approach, the terms A_c and B_c were constructed for different number of resonance modes N relative to the harmonics of ω_0. It is intended to verify how increasing the number of resonance modes affects the tracking performance.

The selected LQR penalty parameters for the Repetitive Controller are $q_1 = 0$, $q_2 = 0$, $q_3 = 0$, $q_4 = 1.0 \cdot 10^8$, $r_1 = 1$ and $q_{ci} = 3.5 \cdot 10^3$ (for $i = 1, \ldots, \tau$). The penalty parameters considered for the Multiple Resonant Controllers are $q_1 = 0$, $q_2 = 0$, $q_3 = 0$, $q_4 = 0$, $r_1 = 5.0 \cdot 10^5$ and $q_{ci} = 1.0 \cdot 10^2$ (for $i = 1, \ldots, 2N$).

The proposed scenario for acquiring results is to initiate the ball position regulation with a simple Proportional-Derivative (PD) Controller and then switch to the Repetitive or Multiple Resonant Controllers. The initial PD is useful to provide a basis for comparison with the other strategies and initialization of the system past errors.

This PD controller was implemented by simply feedbacking the ball position error and the ball velocity state x_2 with the control law:

$$u(k) = -K_{pd} \begin{bmatrix} \hat{x}_1(k) - r(k) \\ \hat{x}_2(k) \end{bmatrix},$$

where the values of the control feedback gains

$$K_{pd} = [0.0054 \quad 0.0024]$$

were found with a similar LQR methodology described to synthesize the Repetitive and Multiple Resonant Controllers.

In order to estimate the unmeasured states x_2, x_3 and x_4 required for Repetitive and Resonant implementations, a linear state observer was developed with the equations

$$\begin{aligned} \hat{x}^*(k) &= A\,\hat{x}(k-1) + B\,u(k-1)\,, \\ \hat{x}(k) &= \hat{x}^*(k) + L\Big(y(k) - C\,\hat{x}^*(k)\Big)\,, \end{aligned}$$

where the values of the observer gain vector

$$L = [0.566 \quad 6.837 \quad -0.011 \quad 0.092\,]'$$

were found with the Linear Quadratic Estimator (LQE) method [Dorato et al., 1995], (also called Static Kalman Filter) that minimizes the estimation error based on the covariances of the system model and output measurements.

4.2 Experimental Results

The subsequent graphics in Figures 3 to 5 present the results achieved with the experiment. The comparative analysis and all the plotted variables refers to the ball X position regulation, since the obtained results for the Y dimension were similar. Note that the sphere position is given in percentage of the plate width, which is 0.28 m.

The plot on Figure 3 shows the effect on the steady-state waveform when increasing the number of resonant modes relative to the harmonics. The unique mode controller presented a sinusoidal like wave form in phase with the reference. As we increase the number of compensated harmonics, the output becomes closer to the reference. It was experimentally observed that increasing the number of modes past four harmonics does not produce practical benefits on the response. So further comparisons were conducted with a four-mode Multiple Resonant, assumed to be the best Resonant implementation on the experiment.

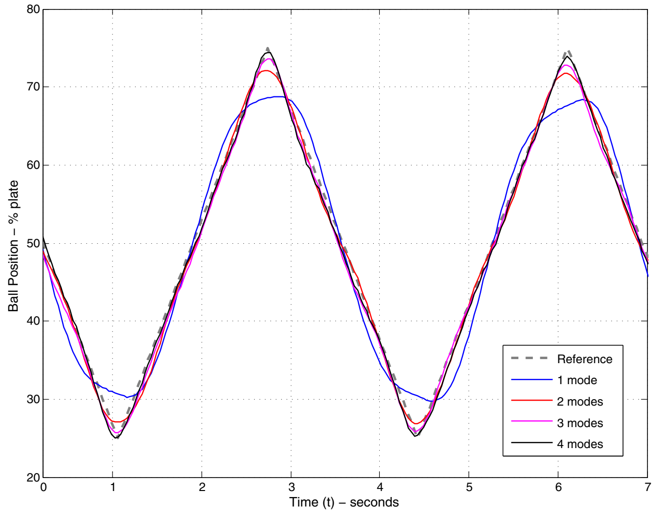

Fig. 3. Comparison of the steady-state response for different resonant controllers.

On Figure 4, it is possible to compare the steady-state response of the sphere position for all there different strategies implemented. The PD response provides a basis result, showing the tracking error for a periodic reference with no internal model on the control loop. Then it possible to compare the steady-state results between the Repetitive Controller and the best Resonant Controller. These two controllers presented very similar responses matching closely to the desired trajectory.

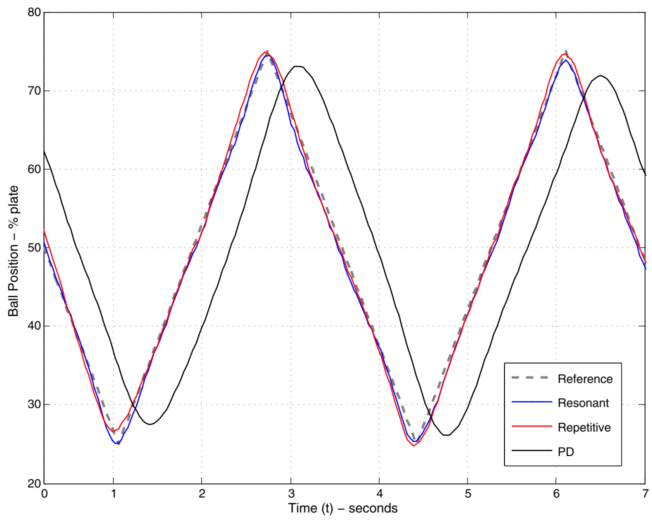

Fig. 4. Comparison of steady-state response between different control strategies.

Table 3 represents the previous results by evaluating the Root Means Square (RMS) of the steady-state error in the control implementations tested. The steady-state period considered for each control scheme is the same presented in Figures 3 and 4.

The last graphic on Figure 5 brings the control input comparison between the implemented controllers running on the steady-state. Since the plate operates with adequate linearity in the tilt range form $-15°$ to $+15°$ it can be stated that no saturation occurred in both executions. The amplitude peeks each controller presented are very

Table 3. RMS of steady-state output error for different controller types.

Controller Type	RMS Error
PD	12.30 %
Resonant (1 mode)	2.64 %
Resonant (2 modes)	1.25 %
Resonant (3 modes)	1.05 %
Resonant (4 modes)	0.69 %
Repetitive	0.75 %

similar, however the Repetitive Controller presented a less oscillatory signal in contrast to the Multiple Resonant Controller.

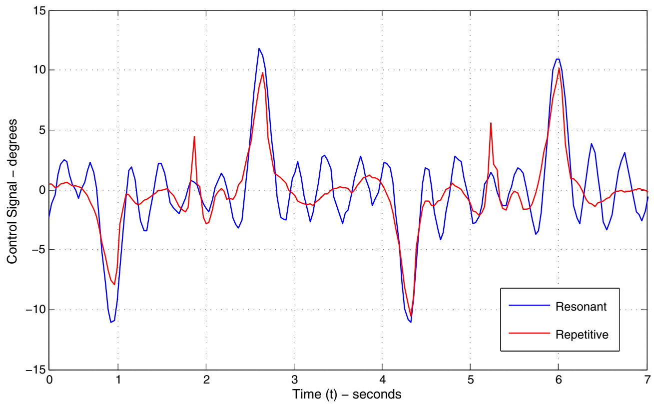

Fig. 5. Comparison of control input between the repetitive and the multiple resonant.

Summarizing, the experimental results showed that increasing modes on a Multiple Resonant Controller produces practical benefits up to finite range. The Repetitive Control presented a very similar steady-state response when compared side-by-side to the best Resonant Controller. The implemented Repetitive Controller however presented a less oscillatory control input.

5. CONCLUSIONS

This work presented a complete design procedure and comparison analysis for two different IMP-based controllers: the Multiple Resonant Controller and the Repetitive Controller. The considered methodology described every aspects to synthesize these architectures, from the state-space equations to the LQR theory for computing the feedback gains. A servo-vision ball and plate balancing system was employed for the practical evaluation of the proposed control methodologies. The article presented the complete plant modeling procedure and setup parameters for the application. Based on the experimental results, a comparative study of each implemented strategy was conducted. The main characteristics analyzed were steady-state error and input control signal.

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